Folding Schemes with Privacy Preserving Selective Verification

Joan Boyar & **Simon Erfurth**

University of Southern Denmark

simon@serfurth.dk www.serfurth.dk

@SimonErfurth @SimonSErfurth

Motivating Example: Computation as a Service

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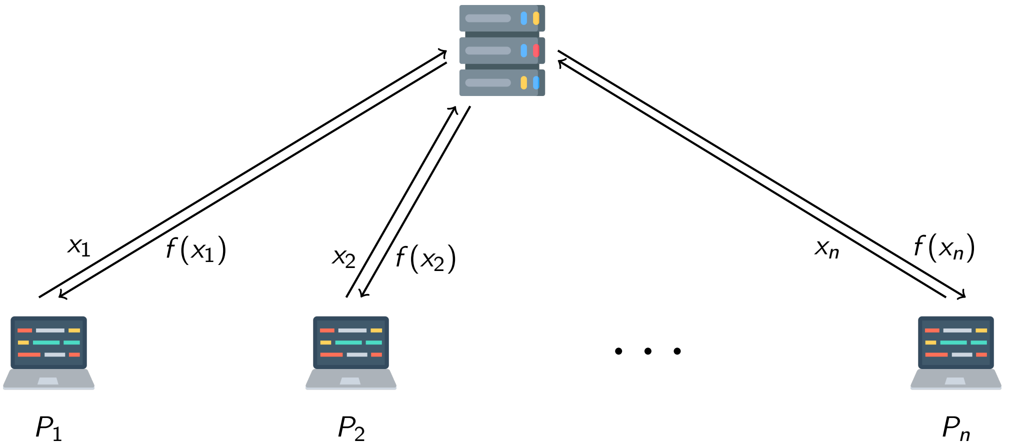
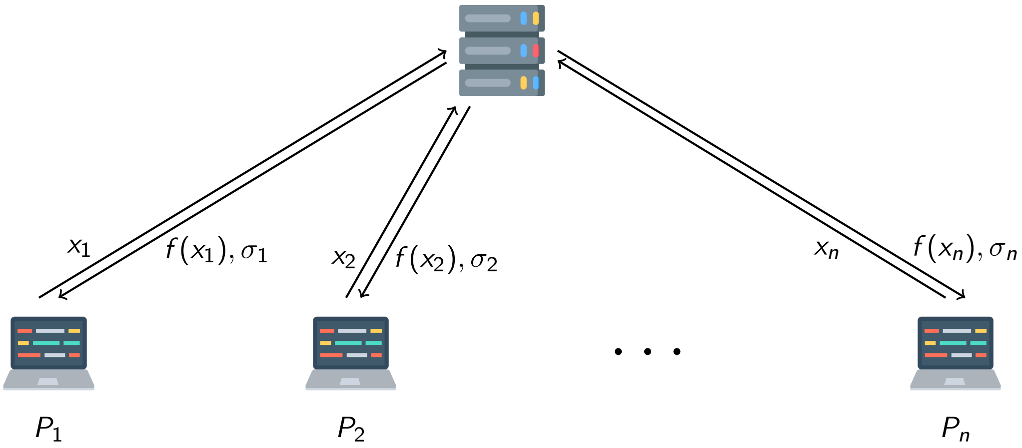


Motivating Example: Verifiable Computation as a Service



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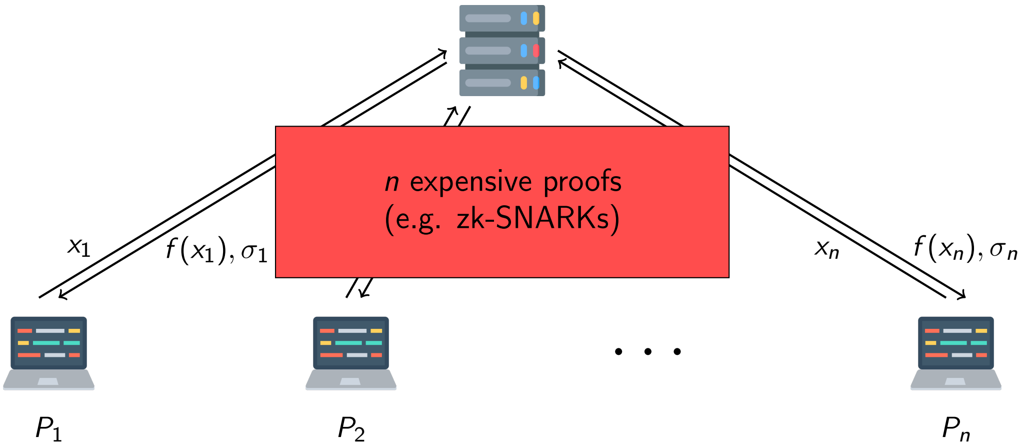

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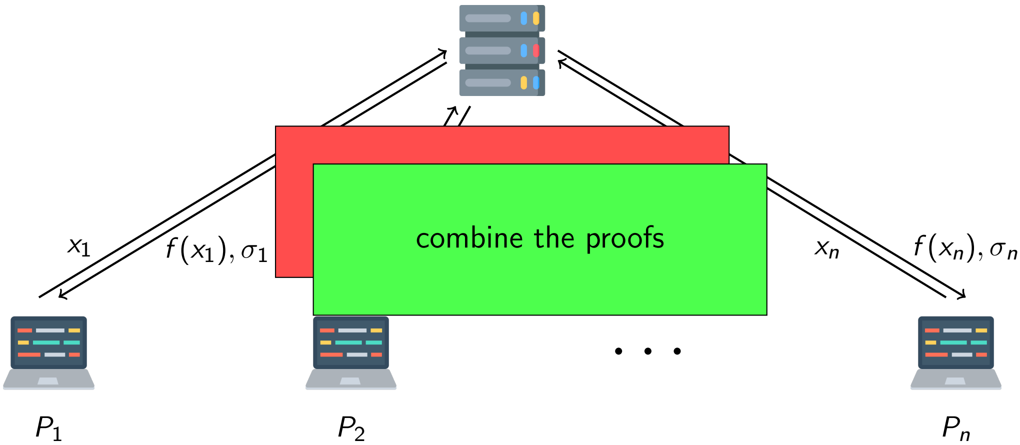

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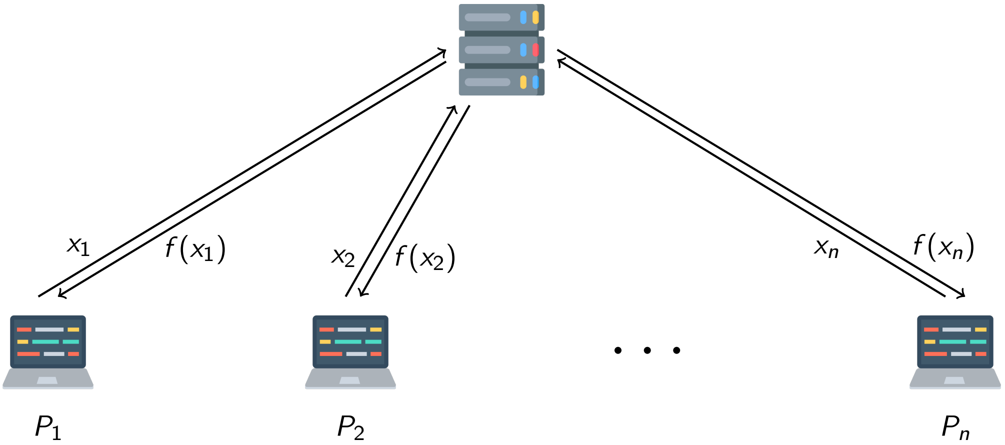

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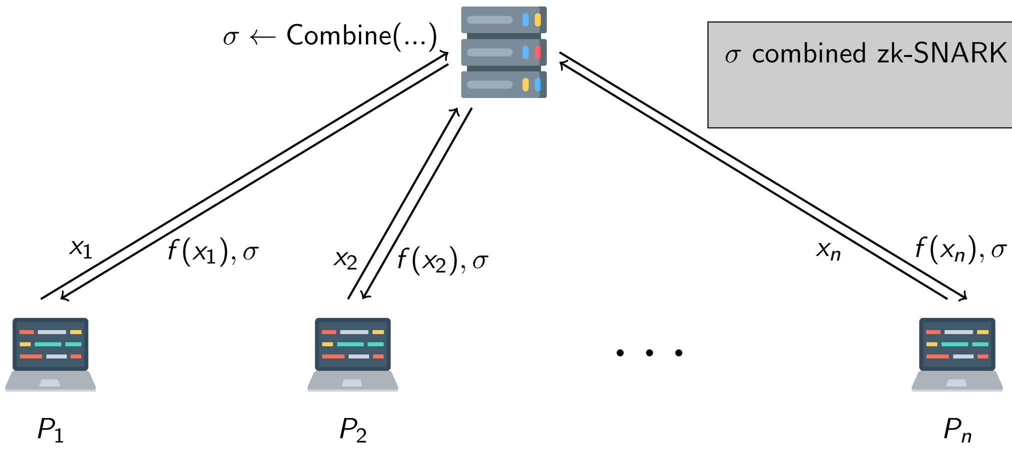

σ combined zk-SNARK

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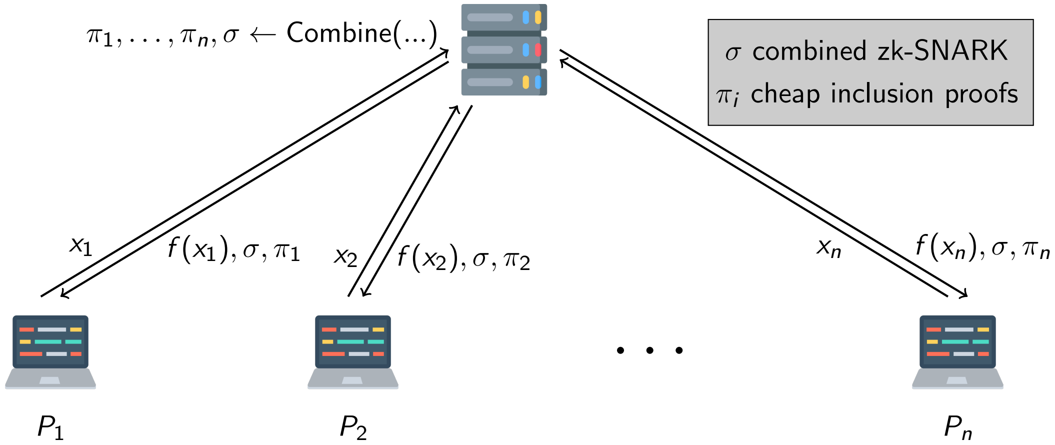

σ combined zk-SNARK
π_i cheap inclusion proofs

Folding Scheme with Selective Verification [KST22]

Folding Scheme

For NP-language $\mathcal{L}$ with relation

$$\mathcal{R} = \{(x, v) \mid v \text{ is a proof that } x \in \mathcal{L}\},$$

folding scheme FS which

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- Combines instances:
  $\text{Fold} : ((x_1, v_1), (x_2, v_2)) \rightarrow (x, v, \pi)$
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- Check statement inclusion
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For $A \in \mathbb{F}^{n \times m}$; $\mathcal{L}_A = \{x \mid \exists v\colon Av = x\}$.

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- $\mathsf{Fold}((x_1, v_1), (x_2, v_2))$:
  $\rho \leftarrow_{\$} \mathbb{F}; \pi = \rho;$

$$x = x_1 + \rho x_2; \qquad v = v_1 + \rho v_2.$$

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  $\rho \leftarrow_{\$} \mathbb{F}; \pi = \rho;$
  $$x = x_1 + \rho x_2; \qquad v = v_1 + \rho v_2.$$
- $\mathsf{FoldVerify}(x_1, x_2, x, \pi)$: check that
  $$x = x_1 + \rho x_2.$$

Folding Scheme: Security

Example

Folding Scheme with Selective Verification

Folding Scheme: Security

- **Completeness**: No Adv. can output input to Fold in $\mathcal{R}$, which gives output not in $\mathcal{R}$ (or invalid folding proof).

Example

- **Completeness**: $(x_1, v_1), (x_2, v_2) \in \mathcal{R}$ then

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Folding Scheme: Security

- **Completeness**: No Adv. can output input to Fold in $\mathcal{R}$, which gives output not in $\mathcal{R}$ (or invalid folding proof).
- **Knowledge Soundness**: From Adv. giving x_1, x_2, x, v, π where $(x, v) \in \mathcal{R}$ and π is accepted, we can extract witness for x_1, x_2.

Example

- **Completeness**: $(x_1, v_1), (x_2, v_2) \in \mathcal{R}$ then

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- **Knowledge Soundness**: Run to get $x, v, \pi = \rho$ and $x', v', \pi' = \rho'$ for same input.

$$\begin{aligned} v =& v_1 + \rho v_2 \\ v' =& v_1 + \rho' v_2 \\ \Rightarrow v_2 =& (\rho' - \rho)^{-1}(v' - v) \end{aligned}$$

From 2-folding to 4-folding

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(x_1, v_1) (x_2, v_2) (x_3, v_3) (x_4, v_4)

Folding Scheme with Selective Verification

From 2-folding to 4-folding

Output of Fold is in $\mathcal{R}$ $\Rightarrow$ **Bootstrapping**

$$\begin{tikzcd}[column sep=small]
& (x', v', \pi') & & & \\
(x_1, v_1) \arrow[ur, "\text{Fold}"] & & (x_2, v_2) \arrow[ul] & (x_3, v_3) & (x_4, v_4)
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From 2-folding to n-folding

Bigger binary tree construction or:

1. $\mathrm{Fold}((x_1, v_1), (x_2, v_2)) \to (x', v', \pi')$
2. $\mathrm{Fold}((x', v'), (x_3, v_3)) \to (x'', v'', \pi'')$ and let $\pi = (\pi', \pi'')$.
3. ...
4. $\mathrm{Fold}((x^{(n-2)}, v^{(n-2)}), (x_n, v_n)) \to (x, v, \pi^{(n-1)})$; $\pi = (\pi', \pi'', \ldots, \pi^{(n-1)})$

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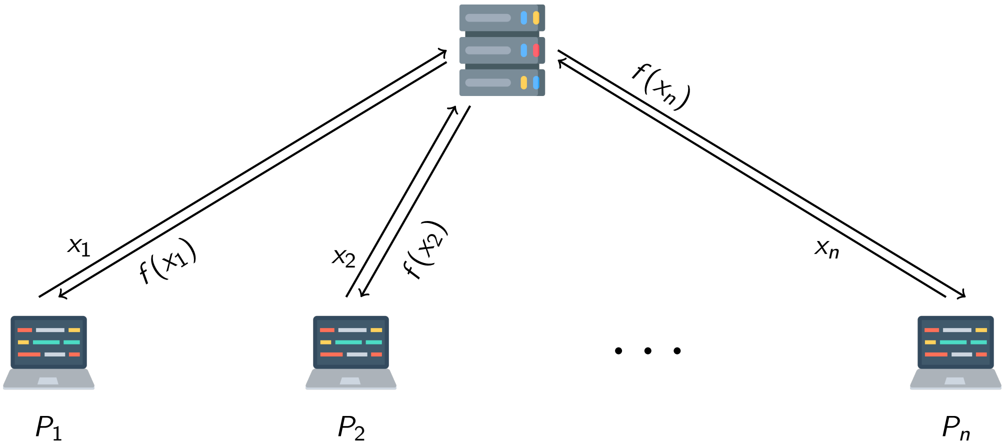

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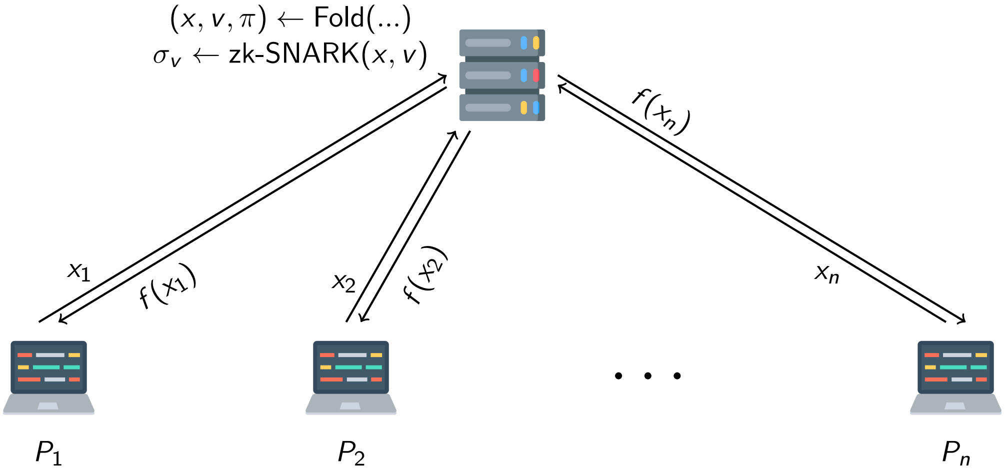

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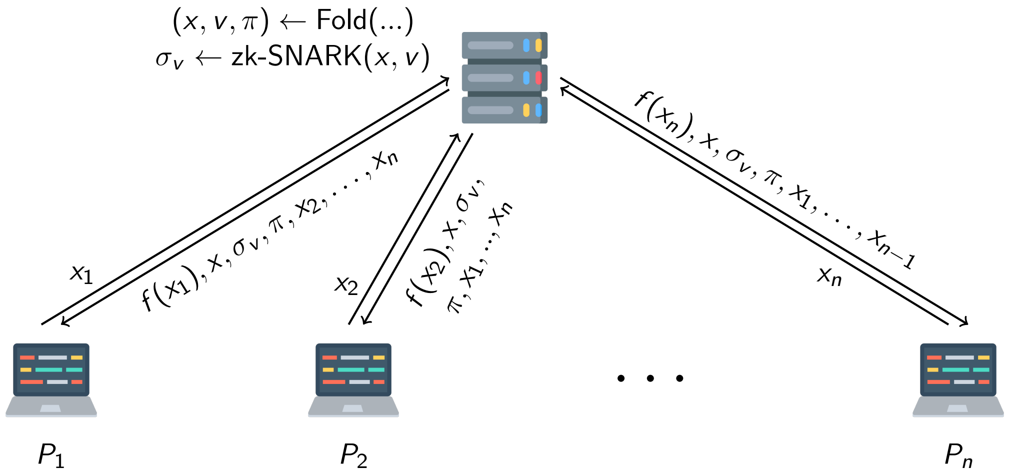

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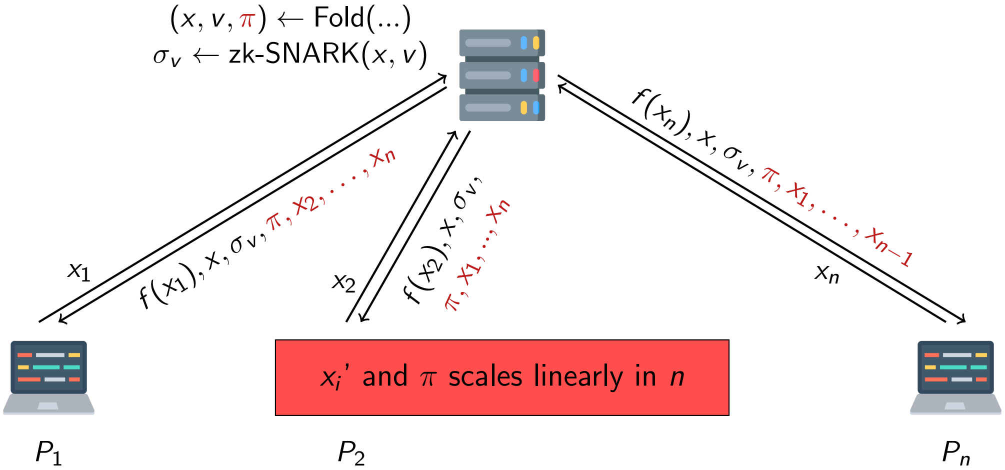

Folding Scheme **with Selective Verification** [RZ23]

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Generate n proofs π_i, each containing $O(\log n)$ folding proofs and statements.

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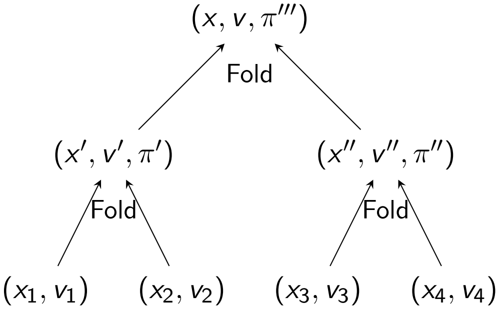

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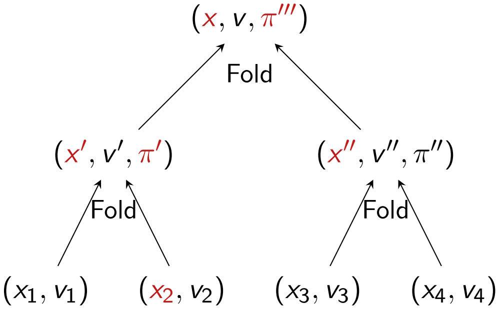

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- $\pi_3 = \{x_4, x'', \pi'', x', x, \pi'''\}$
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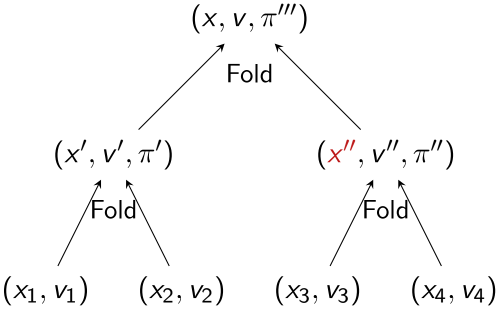

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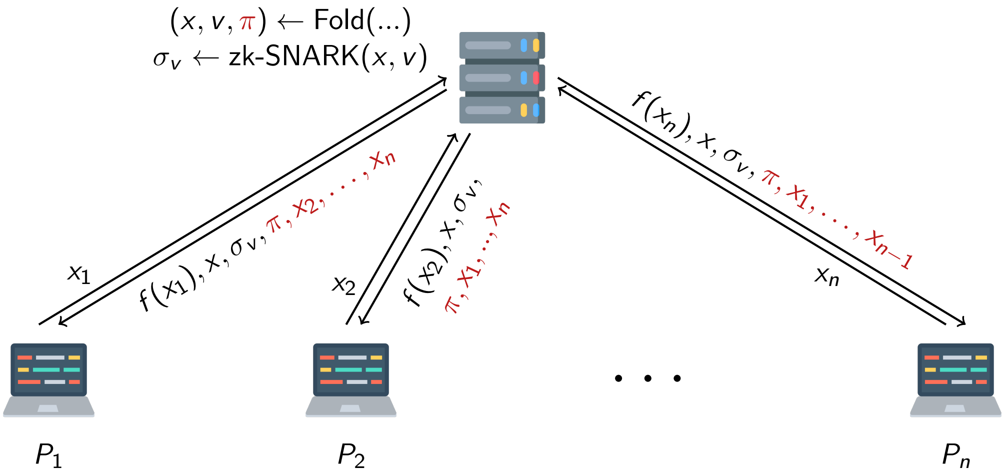

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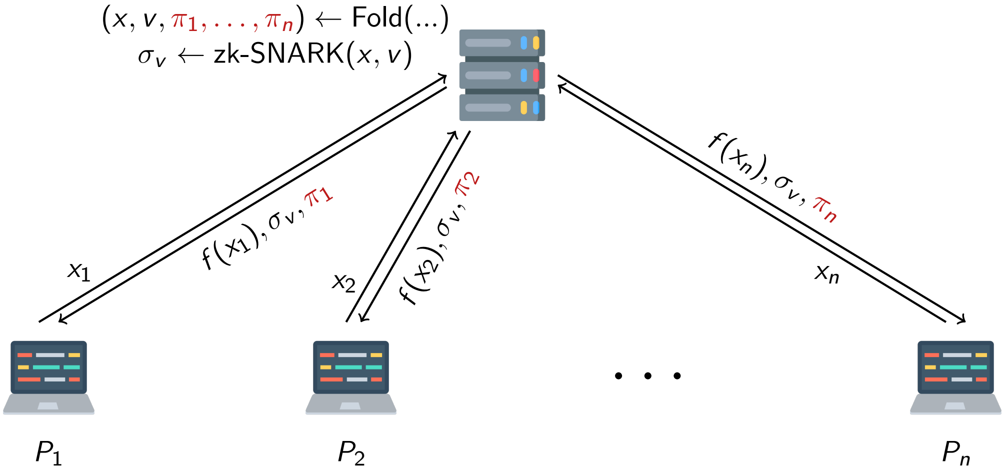

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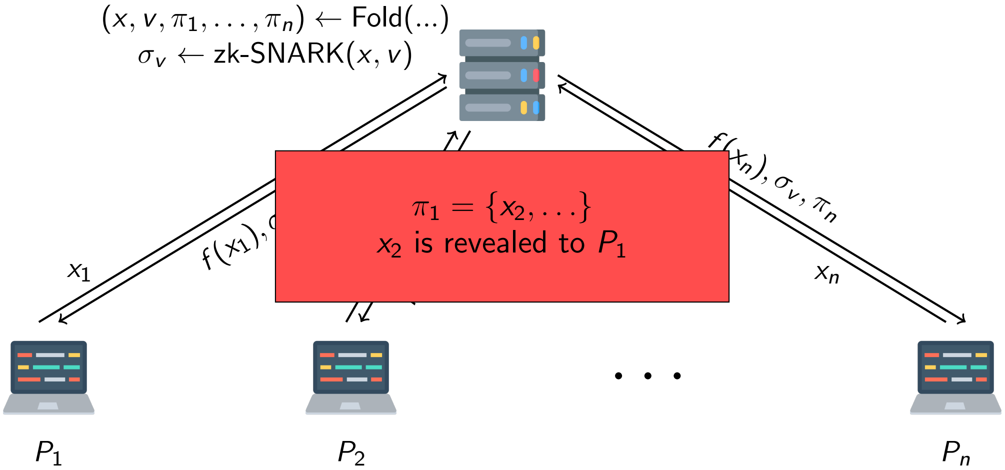

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Folding scheme hiding others' statements.

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Hide each instance (x, v) as another instance (x', v') and generate certificate c that x' hides x. More on these later

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$$
\begin{tikzcd}
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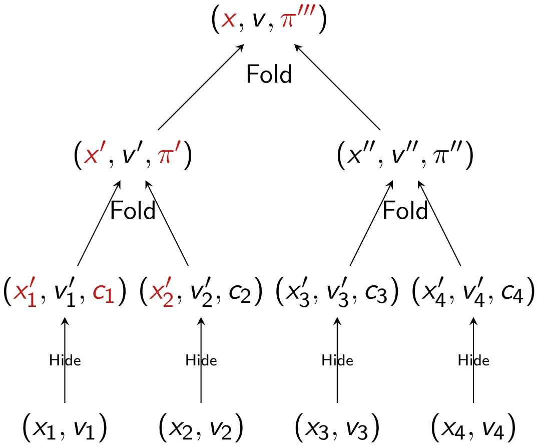

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Security of Privacy Preserving FS

IND-CMA flavor:

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1. Adv choose input with 2 options for entry j

$$(x_1, v_1) \quad (x_2{}^0, v_2^0) \quad (x_3, v_3) \quad (x_4, v_4)$$
$$(x_2{}^1, v_2^1)$$

Security of Privacy Preserving FS

IND-CMA flavor:

1. Adv choose input with 2 options for entry j
2. Entry j chosen at random

$$(x_1, v_1) \quad \begin{matrix}(x_2{}^0, v_2^0)\\(x_2{}^1, v_2^1)\end{matrix} \quad (x_3, v_3) \quad (x_4, v_4)$$

$$b \leftarrow_{\$} \{0, 1\}$$

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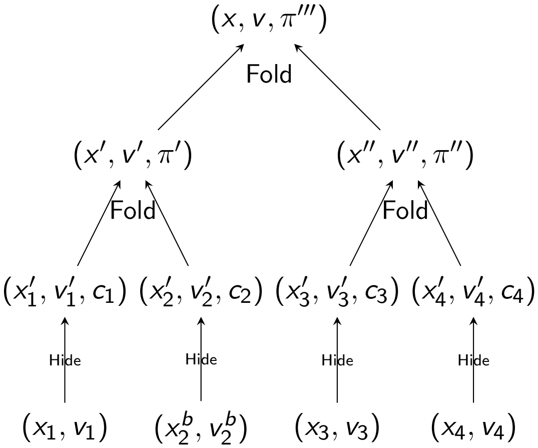

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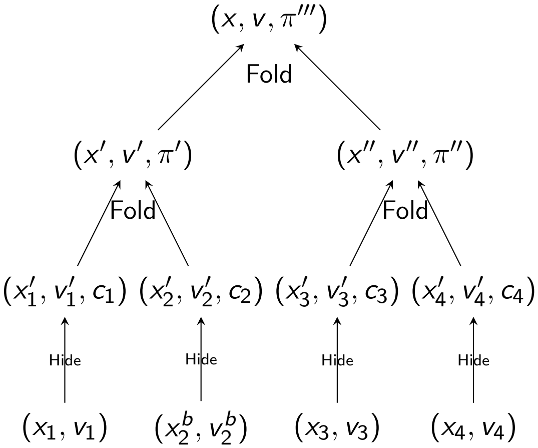
4. Adv chooses index ℓ and receives π_ℓ

$$\mathrm{Adv} \longleftarrow \pi_1$$



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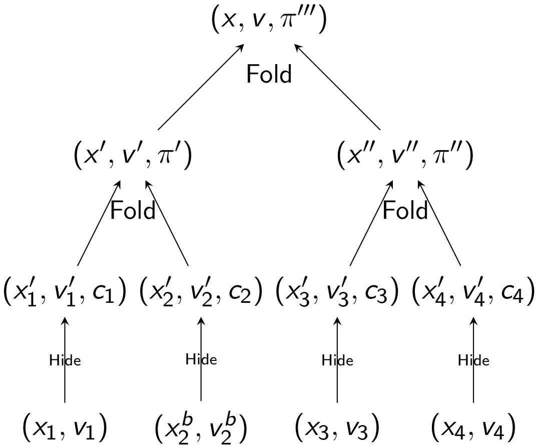
4. Adv chooses index ℓ and receives π_ℓ
5. Guess which (x_j, v_j) was used

$$b' \longleftarrow \text{Adv} \longleftarrow \pi_1$$

Win if $b' = b$



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Folding Scheme with Privacy Preserving Selective Verification [B**E**24]

NP statement hider

$$(x, v) \longrightarrow \text{Hide} \longrightarrow (x', v', c)$$

- Completeness
- Knowledge Soundness
- Hiding

$$(x_b, v_b) \longrightarrow \text{Hide} \longrightarrow (x', v', c)$$

$$b \leftarrow_{\$} \{0, 1\}$$

$$(x', v') \rightarrow \text{Adv} \longrightarrow b'$$

Claim

Composing a Folding Scheme with an NP statement hider gives a Folding Scheme with Privacy Preserving Selective Verification.

NP statement hider: Example

Folding with random instance

To hide (x, v):

1. Generate random instance (x_r, v_r).
2. Fold:
$$(x', v', \pi) \leftarrow \mathrm{Fold}((x, v), (x_r, v_r))$$
3. Output $(x', v', c = (\pi, x_r))$.

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Recall

$$\mathcal{L}_A = \{x \mid \exists v \colon Av = x\}$$

$\mathrm{Fold}((x_1, v_1), (x_2, v_2))$: $\rho \leftarrow_{\$} \mathbb{F}; \pi = \rho;$

$$x = x_1 + \rho x_2; \qquad v = v_1 + \rho v_2.$$

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$\mathsf{Hide}((x, v), (x_r, v_r))$:
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Example is secure

Can Adv distinguish if (x, v) hides (x_1, v_1) or (x_2, v_2)?

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Can Adv distinguish if (x, v) hides (x_1, v_1) or (x_2, v_2)?

Assume (x, v) hides (x_1, v_1) using (x_r, v_r):

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(x, v) is equally likely to hide (x_2, v_2) if there is $(x'_r, v'_r) \in \mathcal{R}_A$ such that:

$$x_2 + \rho' x'_r = x_1 + \rho x_r$$
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$$\begin{aligned} A v'_r &= A(\rho')^{-1}(v_1 + \rho v_r - v_2) \\ &= (\rho')^{-1}(Av_1 + \rho A v_r - Av_2) \\ &= (\rho')^{-1}(x_1 \rho x_r - x_2) \\ &= x'_r \end{aligned}$$

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Theorem

There is a folding scheme with privacy preserving selective verification for $\mathcal{L}_A = \mathrm{Im}(A)$.

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But is this in $\mathcal{R}_A$?

$$\begin{aligned} Av'_r &= A(\rho')^{-1}(v_1 + \rho v_r - v_2) \\ &= (\rho')^{-1}(Av_1 + \rho A v_r - Av_2) \\ &= (\rho')^{-1}(x_1 \rho x_r - x_2) \\ &= x'_r \end{aligned}$$

Conclusion

NP statement hider

If there is a folding scheme for $\mathcal{L}$, $\mathcal{R}$ supports efficient random sampling, and for any three instances $(x_1, v_1), (x_2, v_2), (x, v) \in \mathcal{R}$ there are equally many ways to fold (x_1, v_1) into (x, v) as there is to fold (x_2, v_2) into (x, v), then there is an NP statement hider for $\mathcal{L}$.

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Privacy Preserving Folding Scheme

As above: There is a Privacy Preserving Folding Scheme for $\mathcal{L}$.

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If there is a folding scheme for $\mathcal{L}$, $\mathcal{R}$ supports efficient random sampling, and for any three instances $(x_1, v_1), (x_2, v_2), (x, v) \in \mathcal{R}$ there are equally many ways to fold (x_1, v_1) into (x, v) as there is to fold (x_2, v_2) into (x, v), then there is an NP statement hider for $\mathcal{L}$.

Privacy Preserving Folding Scheme

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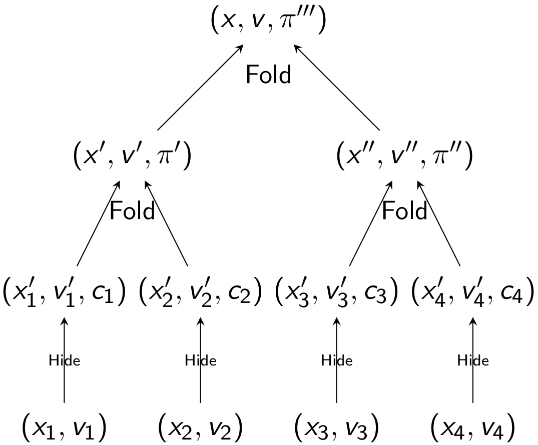
Languages

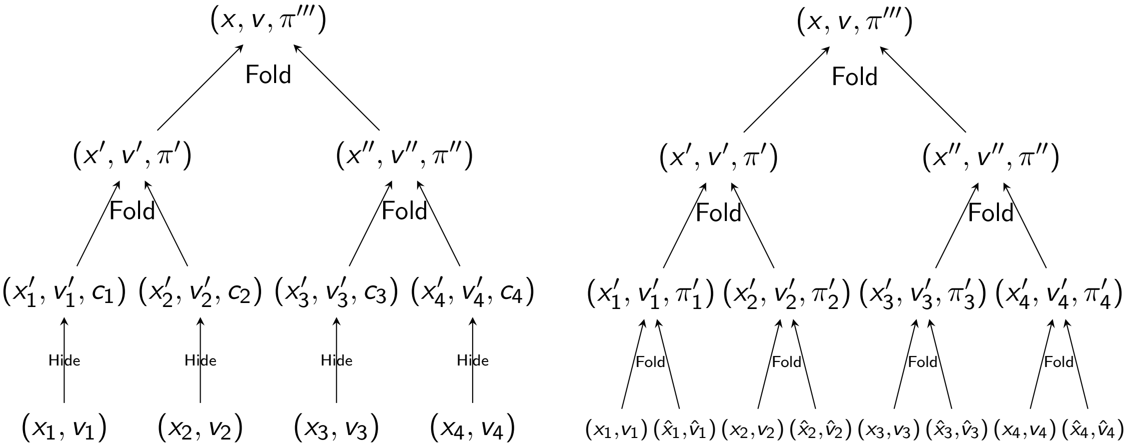
We show that this is satisfied by folding schemes schemes for

- Inner Product Relation of Committed Values [RZ23]
- Committed Relaxed R1CS [KST22]

Folding Scheme with Privacy Preserving Selective Verification [BE24]



Folding Scheme with Privacy Preserving Selective Verification [BE24]



Thank you for listening.

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[BE24]

References

[BE24] Joan Boyar and Simon Erfurth. *Folding Schemes with Privacy Preserving Selective Verification*. Cryptology ePrint Archive, Paper 2024/1530. 2024. URL: `https://eprint.iacr.org/2024/1530`.

[KST22] Abhiram Kothapalli, Srinath T. V. Setty, and Ioanna Tzialla. "Nova: Recursive Zero-Knowledge Arguments from Folding Schemes". In: *Advances in Cryptology - CRYPTO 2022*. Vol. 13510. Lecture Notes in Computer Science. Springer, 2022, pp. 359–388. DOI: 10.1007/978-3-031-15985-5_13.

[RZ23] Carla Ràfols and Alexandros Zacharakis. "Folding Schemes with Selective Verification". In: *Progress in Cryptology - LATINCRYPT 2023*. Vol. 14168. Lecture Notes in Computer Science. Springer, 2023, pp. 229–248. DOI: 10.1007/978-3-031-44469-2_12.

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