Image Authenticity:
Slightly Homomorphic Digital Signatures and Privacy Preserving Folding Schemes

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Universitat Rovira i Virgili, 20/2 2025

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SDU DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

DDC digital democracy centre

Agenda

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1. Motivation: Tracing the provenance of images.

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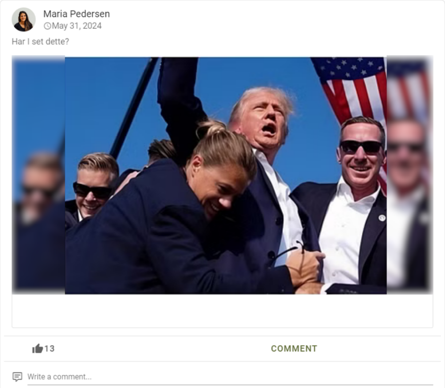
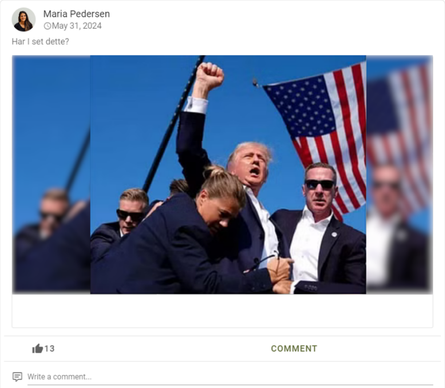
Strengthening Authenticity and Mitigating Misinformation

Image Provenance





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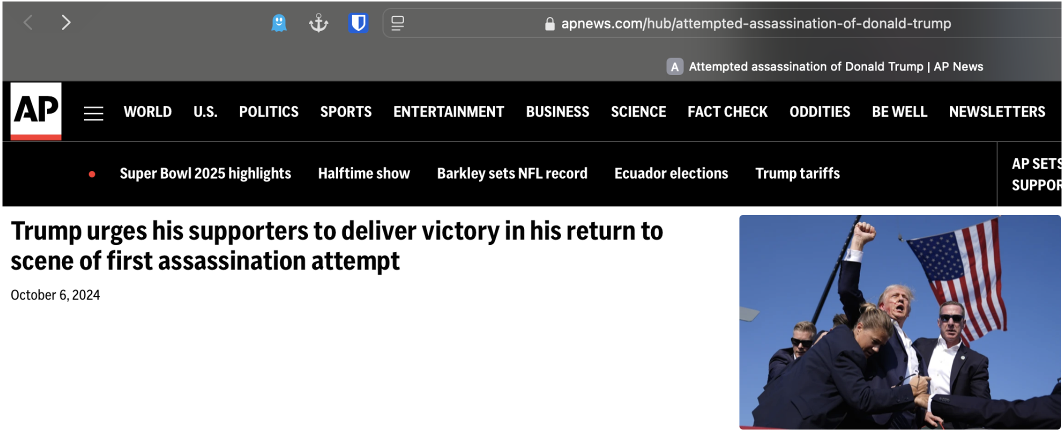

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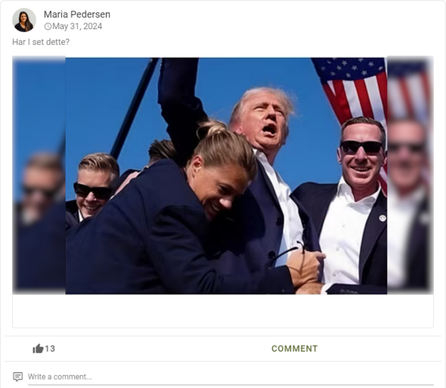

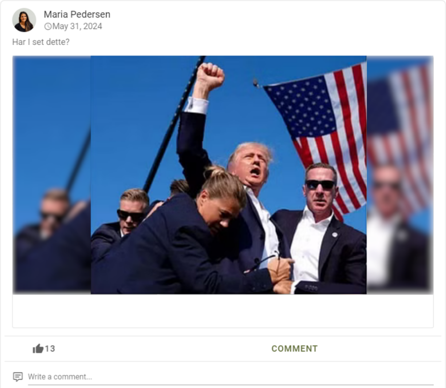

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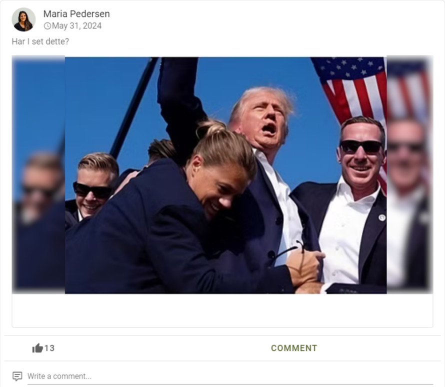

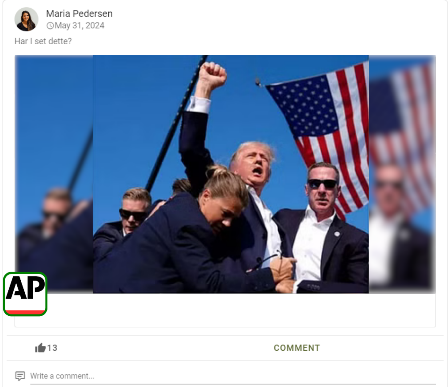

Image Provanance from Digital Signatures

A straight forward solution?

Picture provider signs a signature for the image.

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[PLZ+09]

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Standard digital signatures do not work with (lossy) transformations.

A solution supporting JPEG compression

Slightly Homomorphic Digital Signatures

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P-Homomorphic Signature Scheme

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$(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ regular signature scheme with additional property that

- If $P(m, \{m_1, \ldots, m_i\}) = 1$:
- then *anyone* can extract a signature for m signed with sk from signatures for $m_1, \ldots, m_i$ all signed with sk.

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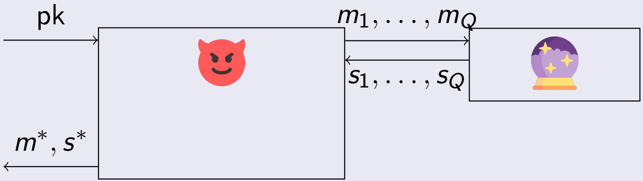
For JPEG compression: Predicate $P(m', M)$ returns 1 if and only if $|M| = 1$ and m' a compression of $m \in M$..

Slightly Homomorphic Digital Signatures

Unforgeability

EUF-CMA

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}()$



if $(\mathsf{Ver}_{\mathsf{pk}}(m^*, s^*) = 1)$ and $(\forall M \subseteq \{m_1, \ldots, m_Q\} \colon P(m^*, M) = 0)$:

$\mathsf{Adv}^{\mathcal{O}_{\mathsf{Sign}_{\mathsf{sk}}(\cdot)}}(\mathsf{pk})$ wins

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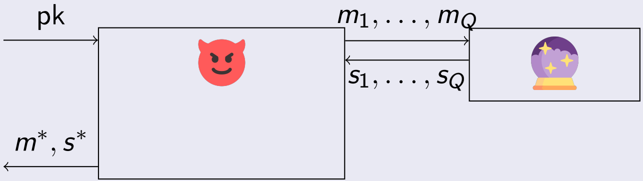

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Towards our construction: JPEG Compression

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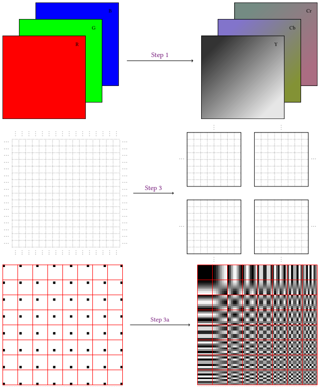
1. RGB to YCbCr color space
2. Optional: Down sample
3. Split into 8×8
   a. Discrete cosine transformation
   b. Quantization
4. Encode using entropy encoder



815	-15	-10	-2	-2	-3	3	3
-3	2	-9	-4	4	-2	-2	3
0	1	-2	-4	-2	-2	2	-1
-3	-1	-3	3	1	0	3	1
0	-1	2	3	-1	1	-1	1
-5	6	1	-1	4	-1	-1	-3
2	1	2	0	0	7	3	-1
0	1	3	-3	-2	3	1	-5

51	-1	-1	0	0	0	0	0
0	0	-1	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
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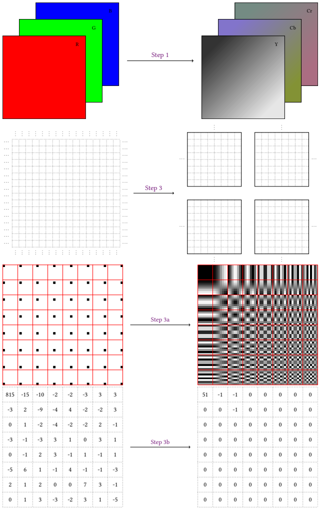

$$\begin{array}{cccccccc} 815 & -15 & -10 & -2 & -2 & -3 & 3 & 3 \\ -3 & 2 & -9 & -4 & 4 & -2 & -2 & 3 \\ 0 & 1 & -2 & -4 & -2 & -2 & 2 & -1 \\ -3 & -1 & -3 & 3 & 1 & 0 & 3 & 1 \\ 0 & -1 & 2 & 3 & -1 & 1 & -1 & 1 \\ -5 & 6 & 1 & -1 & 4 & -1 & -1 & -3 \\ 2 & 1 & 2 & 0 & 0 & 7 & 3 & -1 \\ 0 & 1 & 3 & -3 & -2 & 3 & 1 & -5 \end{array} \xrightarrow{\text{Step 3b}} \begin{array}{cccccccc} 51 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$Q_{50} = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

Construction: 1 Byte

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Observation: Quantization with 2^n is truncation

$$\frac{b_7\, b_6\, b_5\, b_4\, b_3\, b_2\, b_1\, b_0}{2^3}$$

$$\rightsquigarrow b_7\, b_6\, b_5\, b_4\, b_3 xxx$$

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Idea: Provide something instead of xxx?

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$h_1 = H(h_0 \| b_1) \leftarrow h_0 = H(b_0)$

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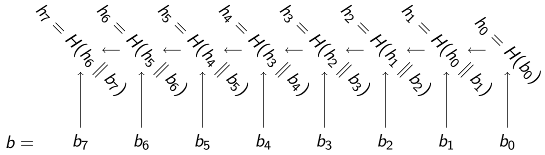

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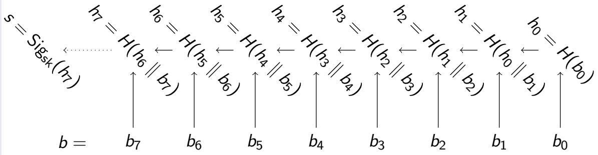

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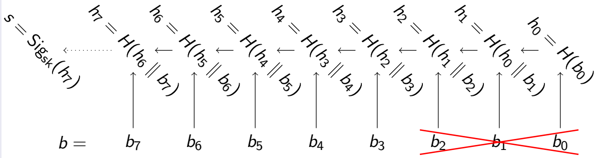

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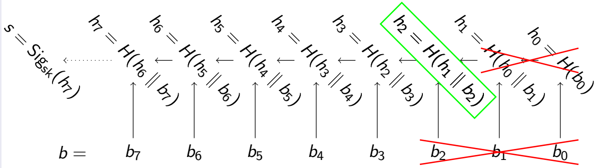

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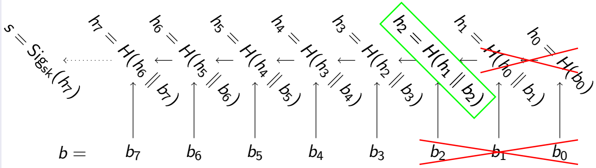
Idea: Provide something instead of *xxx*?

However...

... it is super inefficient!



Construction: Images

But...

- ...each of the 64 DCT coefficients are truncated the same in every 8×8 block
- and images generally have many pixels/blocks

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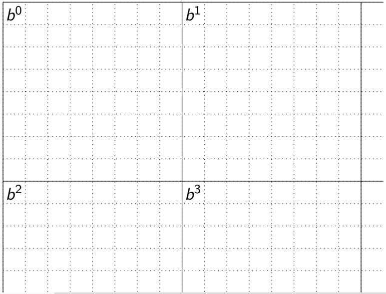
Our solution

- Generate signature by “combining” matching DCT coefficients and then generate hash chains and hash ends together.
- Compress using quantization table with powers of two and update signature to include relevant nodes.

Construction: Images



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b^3 = & b^3_7 & b^3_6 & b^3_5 & b^3_4 & b^3_3 & b^3_2 & b^3_1 & b^3_0 \\
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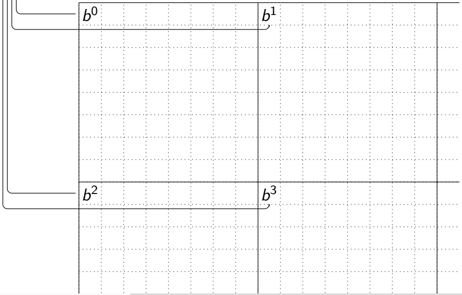
\end{array}$$



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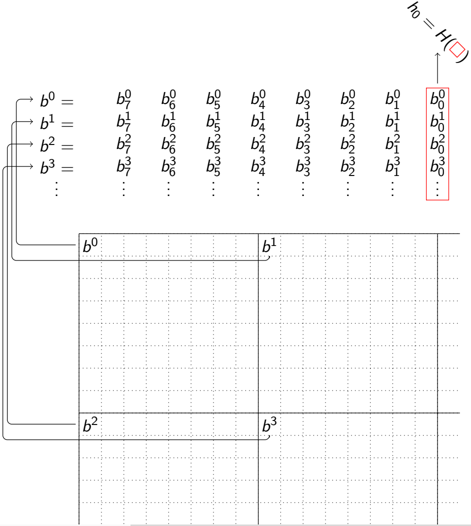

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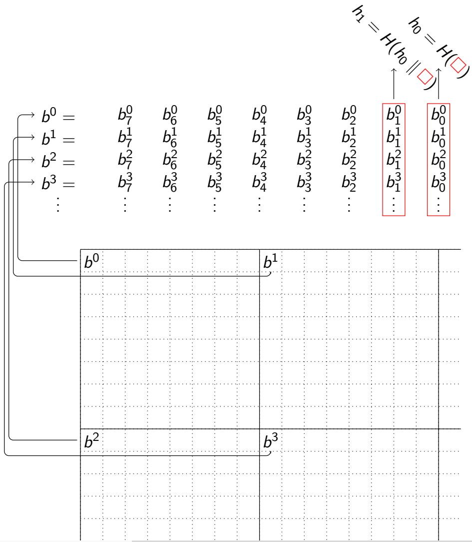

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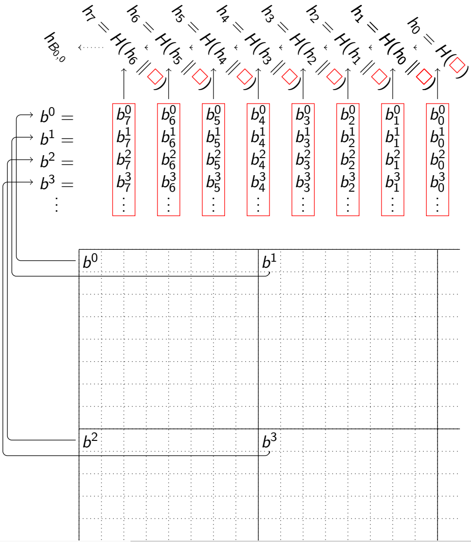

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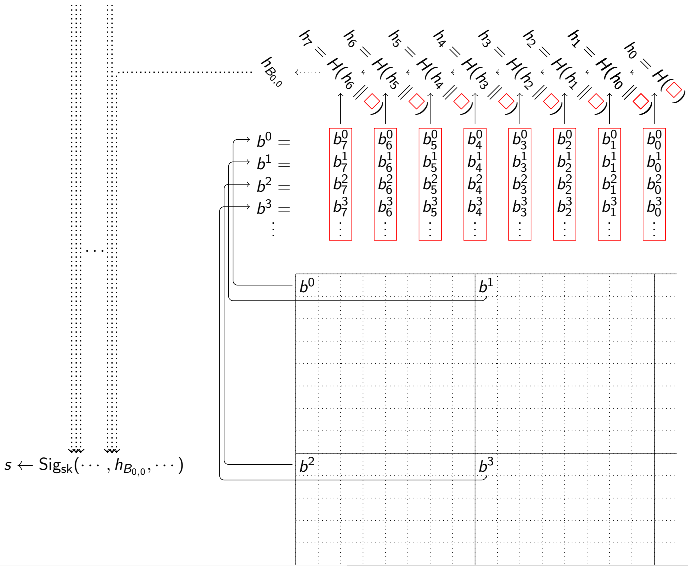

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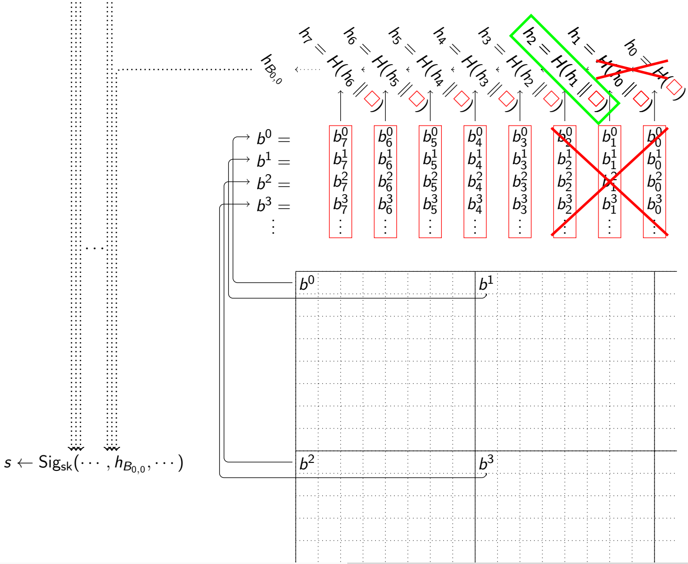

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- $0/1 \leftarrow \mathsf{Verify}_{\mathsf{pk}}(i, s)$: Use i and hashes in s, if any, to find chain ends. Verify with $\mathsf{Verify}^{\mathsf{DS}}$.

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Constructed signature scheme allowing image compression is EUF-CMA.

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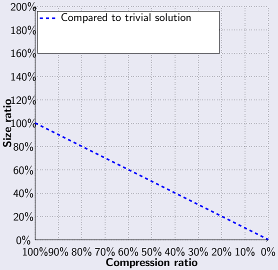
Uncompressed image 2 MB

- - - $|S| = 512$ bits.



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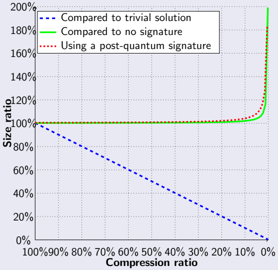

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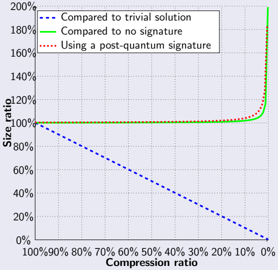

Visual Fidelity

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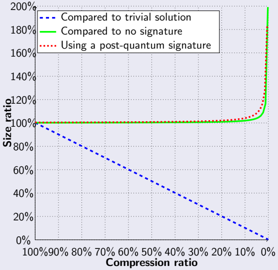


Visual Fidelity

Uncompressed

Classic (QF50)

Powers of two

Security and Performance Claims

Unforgeability

Constructed signature scheme allowing image compression is EUF-CMA.

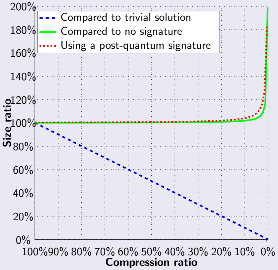
Performance: Signature Size

Uncompressed image 2 MB

- - - $|S| = 512$ bits.

— $|S| = 512$ bits, $|H| = 256$ bits.

······ $|S| = 36760$ bits, $|H| = 256$ bits.



Visual Fidelity

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Powers of two

		Size	MS-SSIM	FSIMc	MSE	PSNR
QF25	**Our tables**	16.0 kB	0.960	0.978	77.526	29.749
	Unmodified	15.1 kB	0.959	0.978	78.900	29.655
	[JWL11] (64)	10.4 kB	0.914	0.936	109.016	28.021
	[JWL11] (32)	20.5 kB	0.961	0.976	46.071	31.706
QF50	**Our tables**	25.4 kB	0.979	0.991	45.831	32.008
	Unmodified	24.4 kB	0.979	0.991	45.910	31.988
	[JWL11] (32)	20.5 kB	0.961	0.976	46.071	31.706
	[JWL11] (16)	36.5 kB	0.983	0.992	18.451	35.605
QF80	**Our tables**	43.9 kB	0.990	0.997	20.256	35.402
	Unmodified	43.4 kB	0.991	0.997	20.432	35.364
	[JWL11] (16)	36.5 kB	0.983	0.992	18.451	35.605
	[JWL11] (8)	60.4 kB	0.993	0.997	7.532	39.439

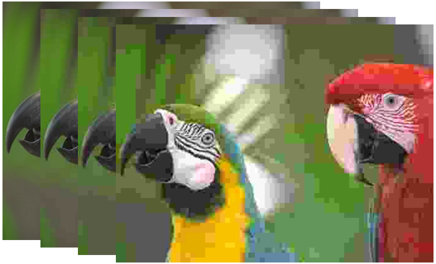
Average over all images in [PLZ+09].

Towards a general solution from SNARKs

Privacy Preserving Folding Schemes

Image Provenance for Other Transformations

[PLZ+09]



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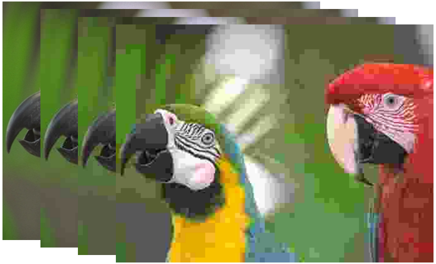
[PLZ+09]



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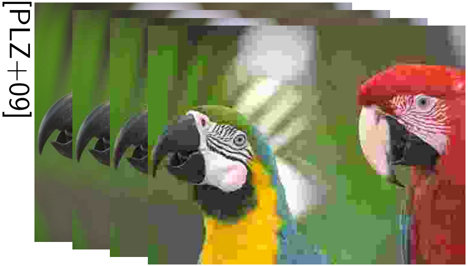


Image Provenance for Other Transformations

[PLZ+09]

Slightly homomorphic signatures support other transformations, but are not practical [JWL11]

Image Provenance for Other Transformations

[PLZ+09]

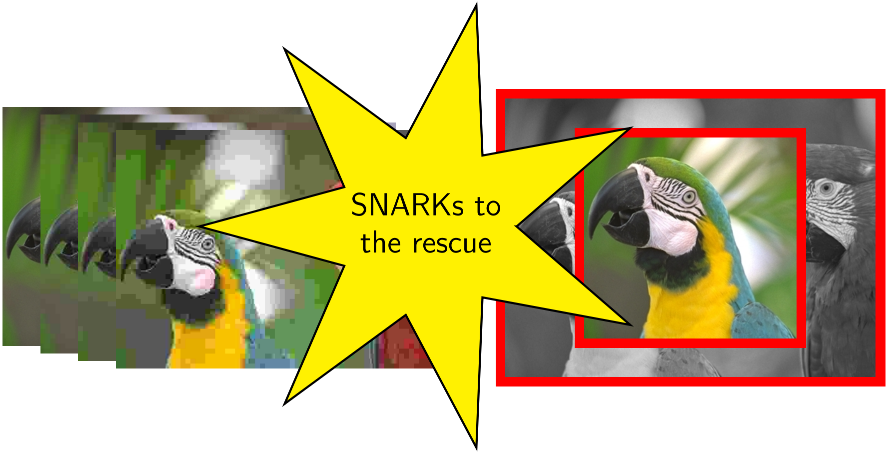
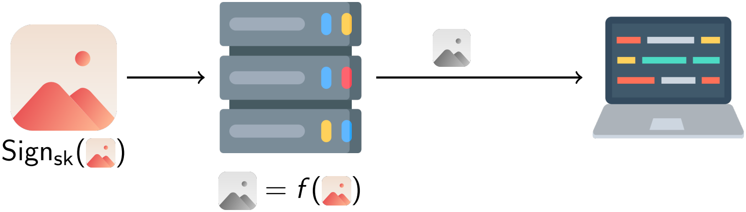


Image Provenance from SNARKs [NT16; DB23; DCB25; DEH25; MVVZ25]



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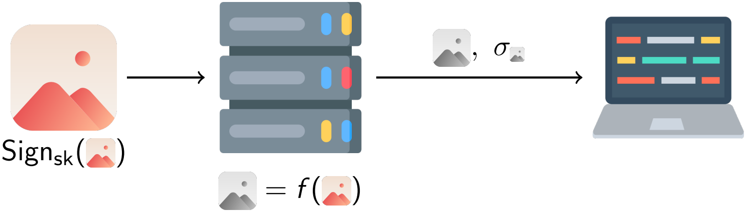

$$\sigma = \begin{cases} \text{I know such that:} \\ 1.\ \text{Sign}_{sk}(\ )\ \text{is valid for} \\ 2.\ \text{is result of } f(\ ) \\ 3.\ \text{Metadata}(\ ) = \text{Metadata}(\ ) \end{cases}$$

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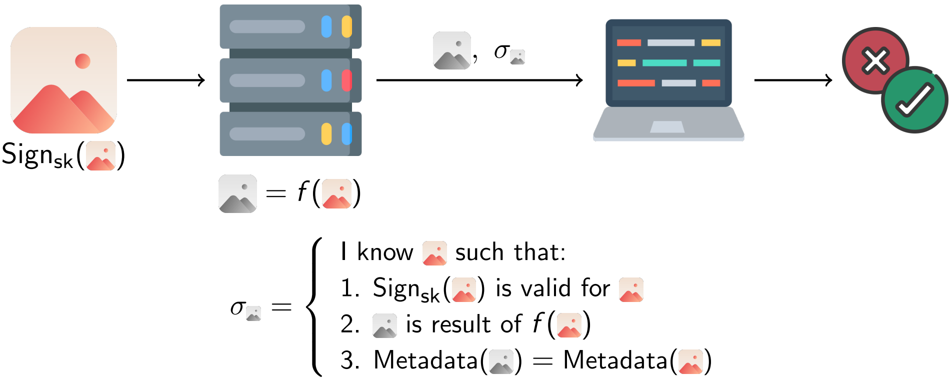

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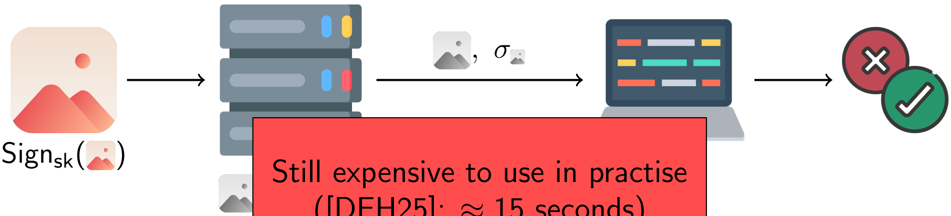

Still expensive to use in practise ([DEH25]: $\approx$ 15 seconds)

$\sigma =$

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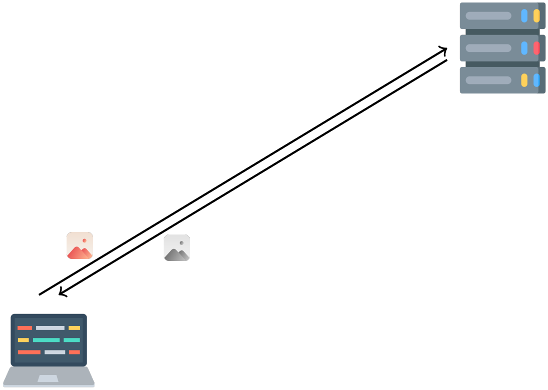
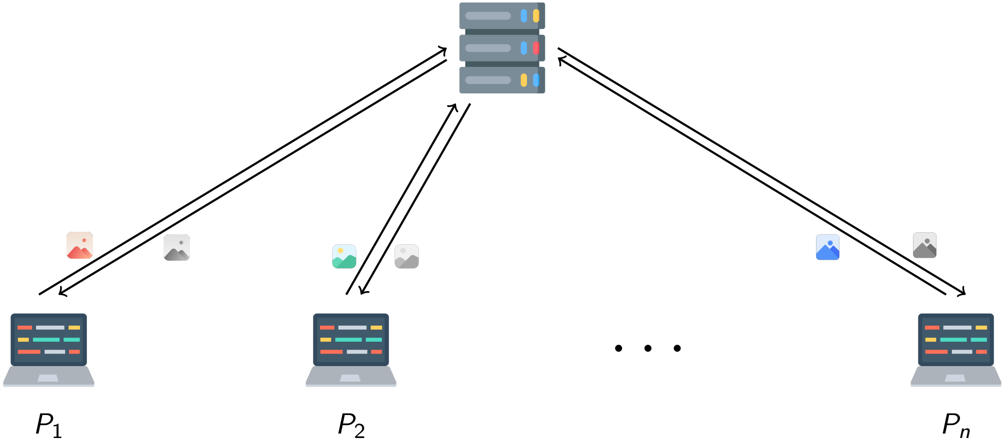


P_1

Restating the Problem: Computation as a Service



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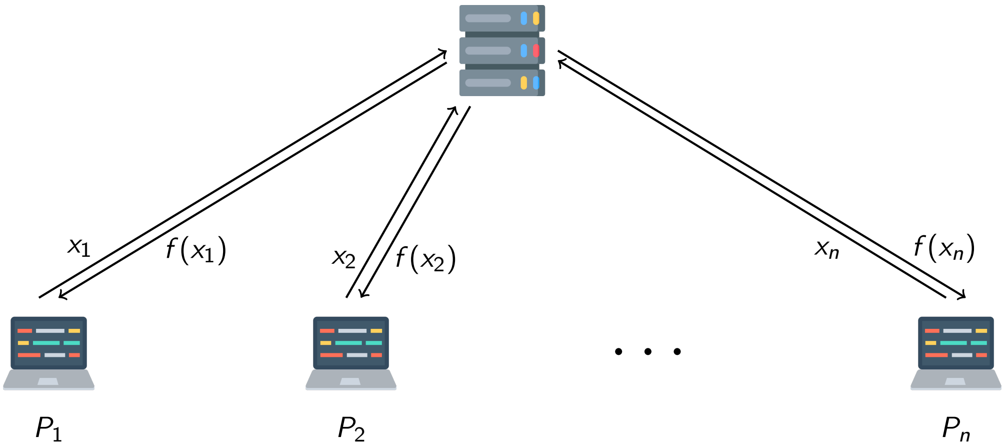
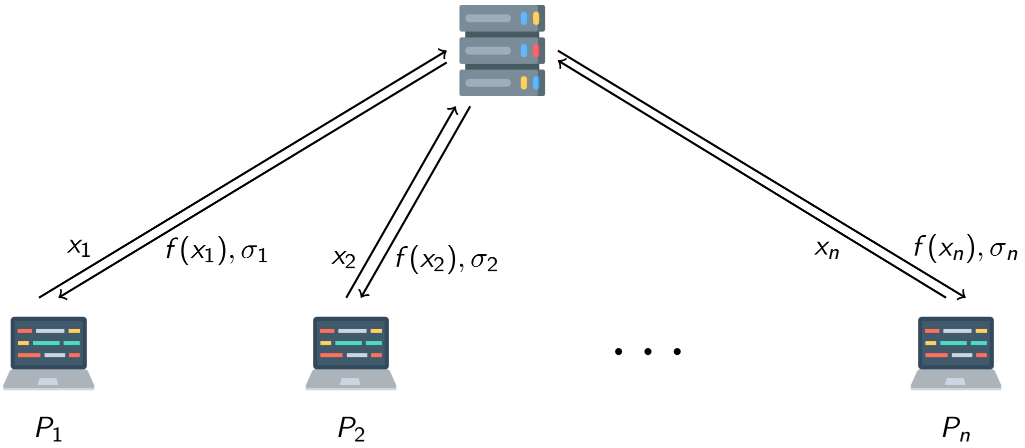


Restating the Problem: Verifiable Computation as a Service



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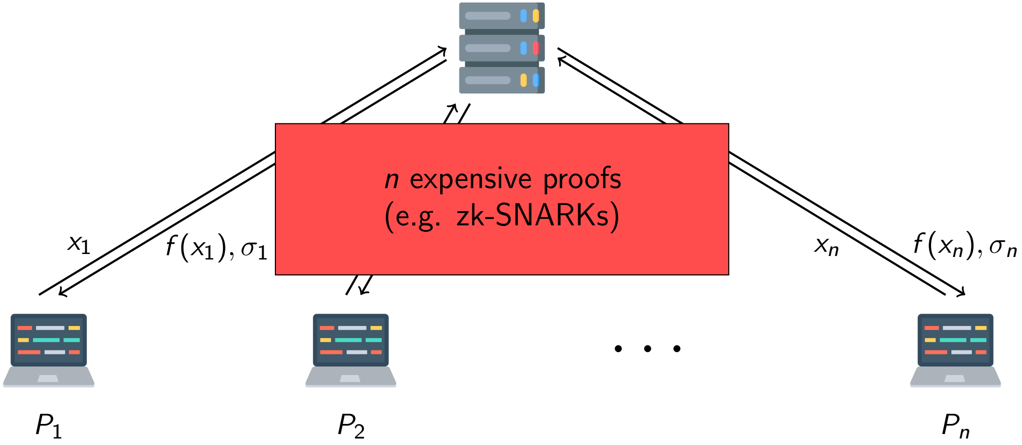

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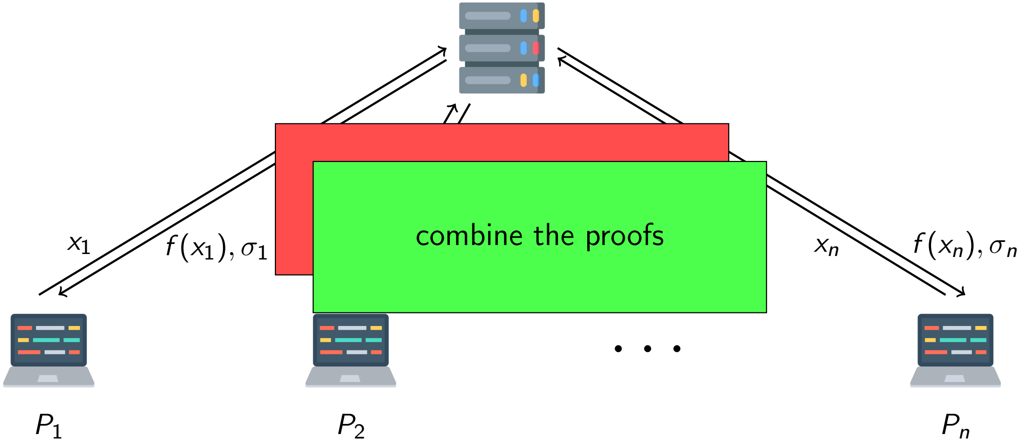

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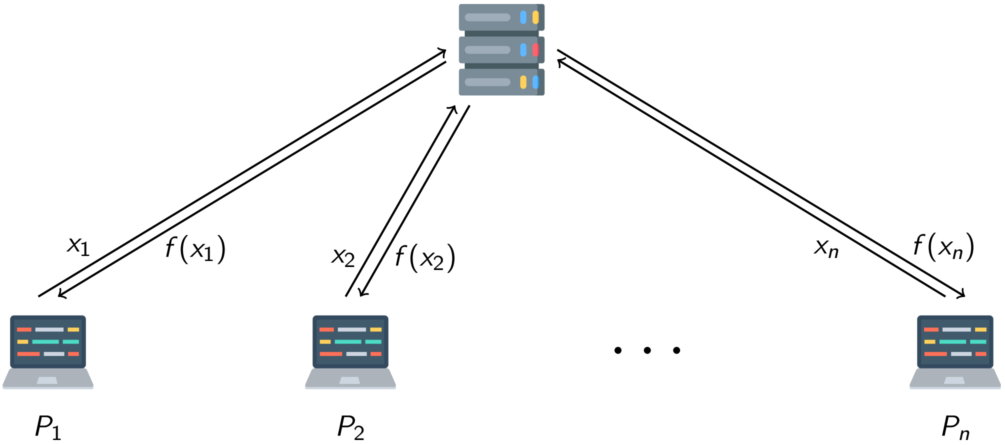

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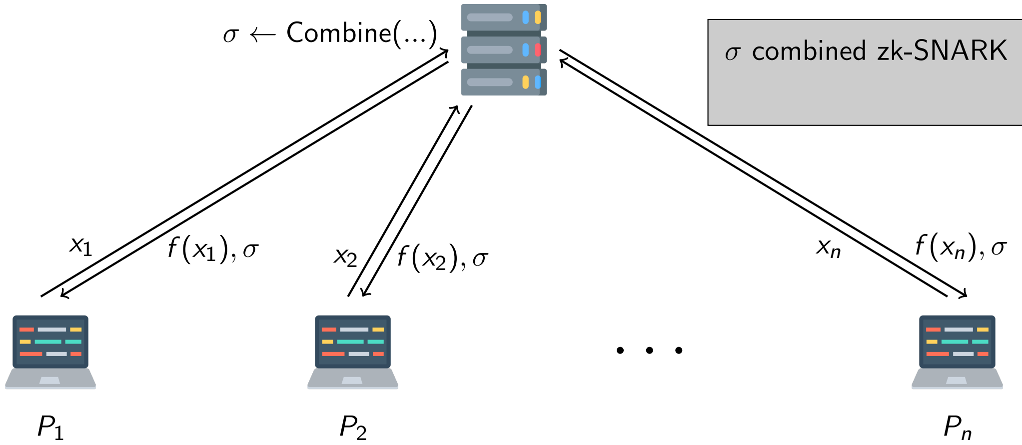

σ combined zk-SNARK

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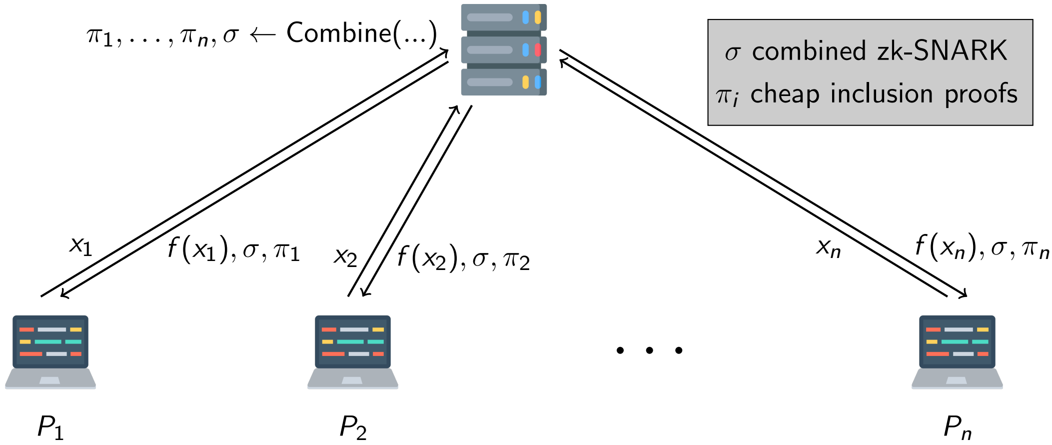

σ combined zk-SNARK

π_i cheap inclusion proofs

Folding Scheme with Selective Verification [KST22]

Folding Scheme

For NP-language $\mathcal{L}$ with relation

$$\mathcal{R} = \{(x, v) \mid v \text{ is a proof that } x \in \mathcal{L}\},$$

folding scheme FS which

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folding scheme FS which

- Combines instances:
  $\mathsf{Fold}\colon ((x_1, v_1), (x_2, v_2)) \to (x, v, \pi)$
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  1 if π is proof that x_1 and x_2 were folded into x

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For $A \in \mathbb{F}^{n\times m}$; $\mathcal{L}_A = \{x \mid \exists v\colon Av = x\}$.

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  $$x = x_1 + \rho x_2; \qquad v = v_1 + \rho v_2.$$

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- $\mathsf{FoldVerify}(x_1, x_2, x, \pi)$: check that
  $$x = x_1 + \rho x_2.$$

Folding Scheme: Security

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- **Completeness**: No Adv. can output input to Fold in $\mathcal{R}$, which gives output not in $\mathcal{R}$ (or invalid folding proof).

Folding Scheme with Selective Verification

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- **Knowledge Soundness**: From Adv. giving x_1, x_2, x, v, π where $(x, v) \in \mathcal{R}$ and π is accepted, we can extract witness for x_1, x_2.

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- **Knowledge Soundness**: Run to get $x, v, \pi = \rho$ and $x', v', \pi' = \rho'$ for same input.

$$\begin{aligned} v =& v_1 + \rho v_2 \\ v' =& v_1 + \rho' v_2 \\ \Rightarrow v_2 =& (\rho' - \rho)^{-1}(v' - v) \end{aligned}$$

Folding Scheme with Selective Verification

From 2-folding to 4-folding

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Output of Fold is in $\mathcal{R} \Rightarrow$ **Bootstrapping**

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Folding Scheme with Selective Verification

From 2-folding to 4-folding

Output of Fold is in $\mathcal{R}$ $\Rightarrow$ **Bootstrapping**

$$(x', v', \pi')$$

Fold

$$(x_1, v_1) \qquad (x_2, v_2) \qquad (x_3, v_3) \qquad (x_4, v_4)$$

Folding Scheme with Selective Verification

From 2-folding to 4-folding

Output of Fold is in $\mathcal{R}$ $\Rightarrow$ **Bootstrapping**

$$
\begin{tikzcd}
 & (x', v', \pi') & & & (x'', v'', \pi'') & \\
(x_1, v_1) \arrow[ur, "\mathsf{Fold}"] & & (x_2, v_2) \arrow[ul] & (x_3, v_3) \arrow[ur, "\mathsf{Fold}"] & & (x_4, v_4) \arrow[ul]
\end{tikzcd}
$$

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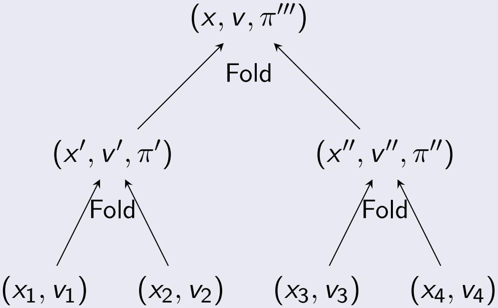

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$$
\begin{tikzcd}
 & & & (x, v, \pi''') & & & \\
 & (x', v', \pi') \arrow[urr, "\text{Fold}"] & & & & (x'', v'', \pi'') \arrow[ull] & \\
(x_1, v_1) \arrow[ur, "\text{Fold}"] & & (x_2, v_2) \arrow[ul] & & (x_3, v_3) \arrow[ur, "\text{Fold}"] & & (x_4, v_4) \arrow[ul]
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$$

$$\pi = (\pi', \pi'', \pi''')$$

Folding Scheme with Selective Verification

From 2-folding to 4-folding

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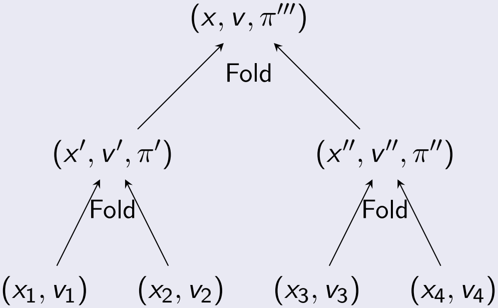

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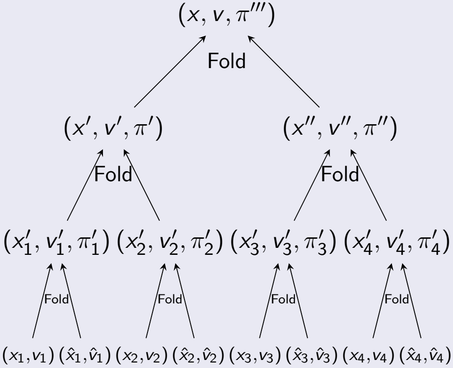
From 2-folding to n-folding

Binary tree with n leaves



Folding Scheme **with Selective Verification** [RZ23]

But...

$|\pi|$ and number of x_i's needed scales linearly in n.

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Idea

Generate n proofs π_i, each containing $O(\log n)$ folding proofs and statements.

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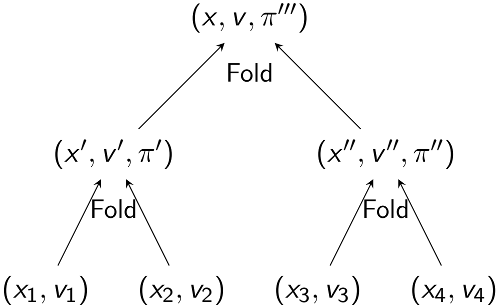

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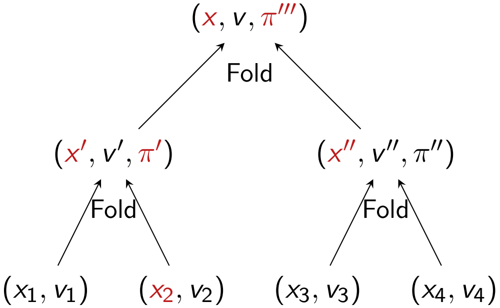
Example

- $\pi_1 = \{x_2, x', \pi', x'', x, \pi'''\}$



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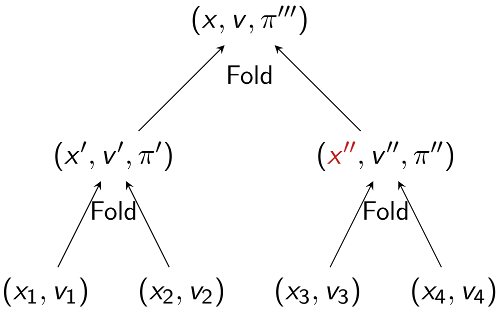
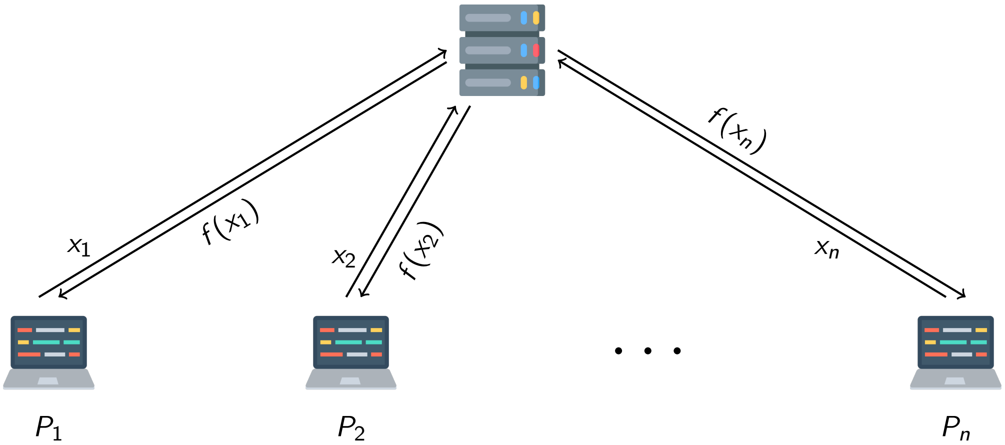


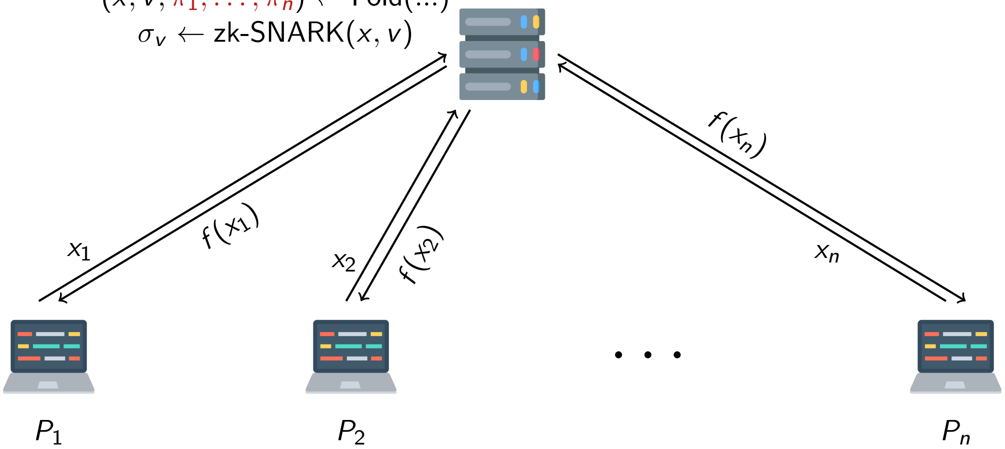
Motivating Example: Verifiable Computation as a Service



Motivating Example: Verifiable Computation as a Service

$(x, v, \pi_1, \ldots, \pi_n) \leftarrow \text{Fold}(\ldots)$

$\sigma_v \leftarrow \text{zk-SNARK}(x, v)$



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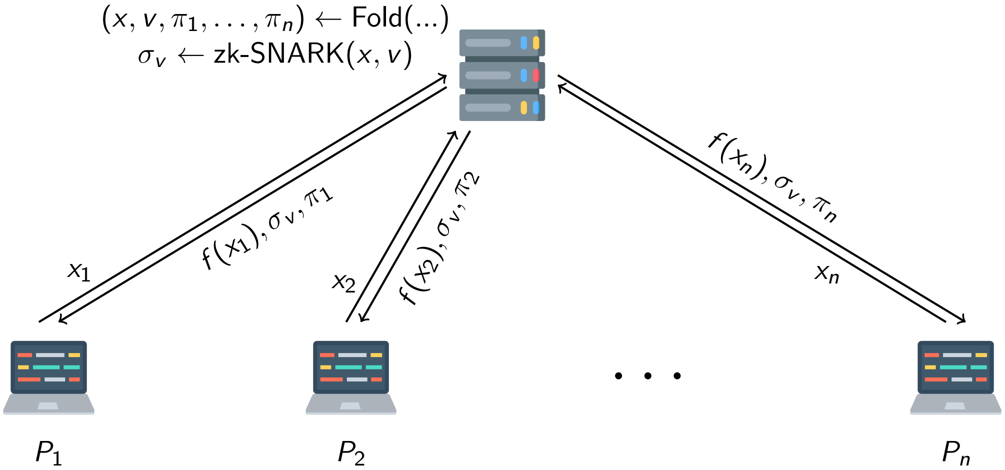

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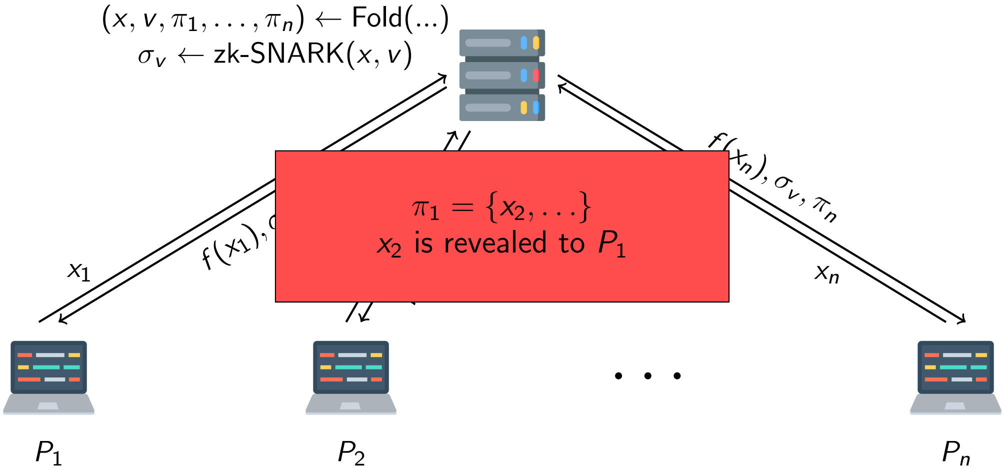

$\pi_1 = \{x_2, \ldots\}$

x_2 is revealed to P_1

Privacy Preserving Folding Scheme [B**E**24]

Idea

Folding scheme hiding others' statements.

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Folding scheme hiding others' statements.

NP-statement hider

Hide each instance (x, v) as another instance (x', v') and generate certificate c that x' hides x. More on these later

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Hide each instance (x, v) as another instance (x', v') and generate certificate c that x' hides x. More on these later

Example

- $\pi_1 = \{x'_1, c_1 \quad\quad\quad\quad\quad\quad \}$
- $\pi_2 = \{x'_2, c_2 \quad\quad\quad\quad\quad\quad \}$
- $\pi_3 = \{x'_3, c_3 \quad\quad\quad\quad\quad\quad \}$
- $\pi_4 = \{x'_4, c_4 \quad\quad\quad\quad\quad\quad \}$

$$
\begin{tikzcd}
(x'_1, v'_1, c_1) & (x'_2, v'_2, c_2) & (x'_3, v'_3, c_3) & (x'_4, v'_4, c_4) \\
(x_1, v_1) \arrow[u, "\text{Hide}"] & (x_2, v_2) \arrow[u, "\text{Hide}"] & (x_3, v_3) \arrow[u, "\text{Hide}"] & (x_4, v_4) \arrow[u, "\text{Hide}"]
\end{tikzcd}
$$

Privacy Preserving Folding Scheme [BE24]

Idea

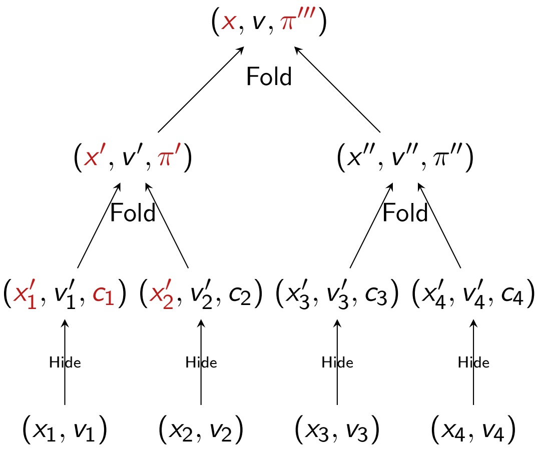
Folding scheme hiding others' statements.

NP-statement hider

Hide each instance (x, v) as another instance (x', v') and generate certificate c that x' hides x. More on these later

Example

- $\pi_1 = \{x_1', c_1, x_2', x', \pi', x, \pi'''\}$
- $\pi_2 = \{x_2', c_2, x_1', x', \pi', x, \pi'''\}$
- $\pi_3 = \{x_3', c_3, x_4', x'', \pi'', x, \pi'''\}$
- $\pi_4 = \{x_4', c_4, x_3', x'', \pi'', x, \pi'''\}$



Privacy Preserving Folding Scheme [B**E**24]

Security of Privacy Preserving FS

IND-CMA flavor:

Privacy Preserving Folding Scheme [BE24]

Security of Privacy Preserving FS

IND-CMA flavor:

1. Adv choose input with 2 options for entry j

$$(x_1, v_1) \quad \begin{matrix} ({x_2}^0, v_2^0) \\ ({x_2}^1, v_2^1) \end{matrix} \quad (x_3, v_3) \quad (x_4, v_4)$$

Privacy Preserving Folding Scheme [BE24]

Security of Privacy Preserving FS

IND-CMA flavor:

1. Adv choose input with 2 options for entry j
2. Entry j chosen at random

$$(x_1, v_1) \quad \begin{matrix} ({x_2}^0, v_2^0) \\ ({x_2}^1, v_2^1) \end{matrix} \quad (x_3, v_3) \quad (x_4, v_4)$$

$$b \leftarrow_\$ \{0, 1\}$$

Privacy Preserving Folding Scheme [B**E**24]

Security of Privacy Preserving FS

IND-CMA flavor:

1. Adv choose input with 2 options for entry j
2. Entry j chosen at random

$$(x_1, v_1) \quad (x_2^b, v_2^b) \quad (x_3, v_3) \quad (x_4, v_4)$$

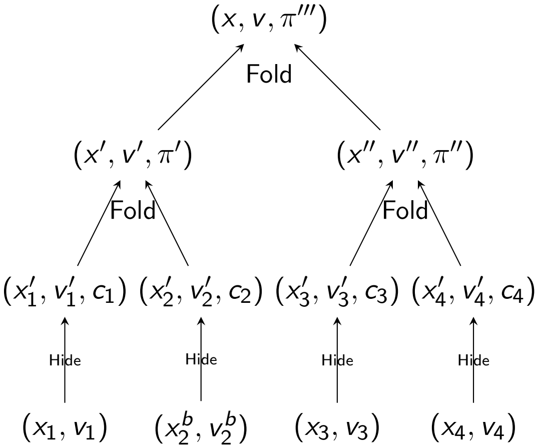
$$b \leftarrow_{\$} \{0, 1\}$$

Privacy Preserving Folding Scheme [BE24]

Security of Privacy Preserving FS

IND-CMA flavor:

1. Adv choose input with 2 options for entry j
2. Entry j chosen at random
3. Everything is hidden and folded



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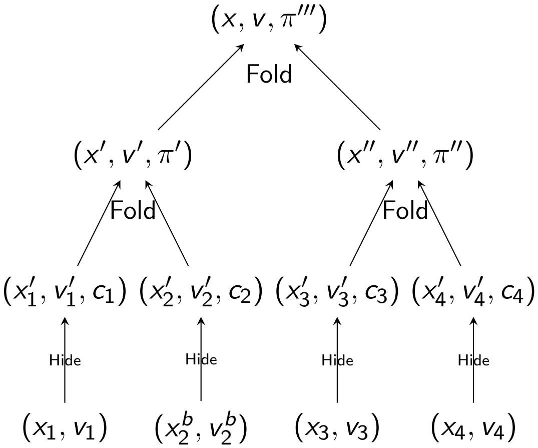
4. Adv chooses index ℓ and receives π_ℓ



$$\mathrm{Adv} \longleftarrow \pi_1$$

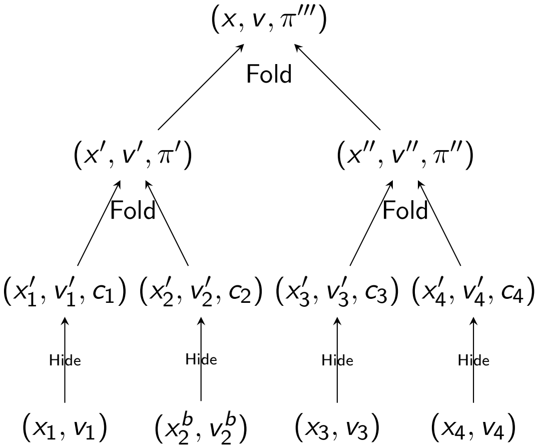
$$b \leftarrow_{\$} \{0, 1\}$$

Privacy Preserving Folding Scheme [BE24]

Security of Privacy Preserving FS

IND-CMA flavor:

1. Adv choose input with 2 options for entry j
2. Entry j chosen at random
3. Everything is hidden and folded
4. Adv chooses index ℓ and receives π_ℓ
5. Guess which (x_j, v_j) was used



$$b' \longleftarrow \text{Adv} \longleftarrow \pi_1$$

$$b \leftarrow_{\$} \{0, 1\}$$

Win if $b' = b$

Privacy Preserving Folding Scheme [BE24]

NP-statement hider

$$(x, v) \longrightarrow \text{Hide} \longrightarrow (x', v', c)$$

Privacy Preserving Folding Scheme [B**E**24]

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$$(x, v) \longrightarrow \text{Hide} \longrightarrow (x', v', c)$$

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- Knowledge Soundness

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  (x_0, v_0)
  (x_1, v_1)

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$$(x', v') \rightarrow \text{Adv}$$

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Claim

Composing a Folding Scheme with an NP-statement hider gives a Privacy Preserving Folding Scheme.

NP-statement hider: Example

Folding with random instance

To hide (x, v):

1. Generate random instance (x_r, v_r).
2. Fold:
$$(x', v', \pi) \leftarrow \mathrm{Fold}((x, v), (x_r, v_r))$$
3. Output $(x', v', c = (\pi, x_r))$.

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Recall

$$\mathcal{L}_A = \{x \mid \exists v \colon Av = x\}$$

$\mathrm{Fold}((x_1, v_1), (x_2, v_2))$: $\rho \leftarrow_{\$} \mathbb{F}; \pi = \rho;$

$$x = x_1 + \rho x_2; \qquad v = v_1 + \rho v_2.$$

Example

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Example

1. Generate random instance in $\mathcal{R}$ as
$$v_r \leftarrow_\$ \mathbb{F}^m; \qquad x_r = Av_r.$$

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1. Generate random instance in $\mathcal{R}$ as
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$\mathsf{Hide}((x, v), (x_r, v_r))$:
$$\rho \leftarrow_\$ \mathbb{F}$$
$$x' = x_1 + \rho x_r$$
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NP-statement hider: Example

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3. Output $(x', v', c = (\rho, x_r))$.

NP-statement hider: Example

Example is secure

Can Adv distinguish if (x, v) hides (x_1, v_1) or (x_2, v_2)?

NP-statement hider: Example

Example is secure

Can Adv distinguish if (x, v) hides (x_1, v_1) or (x_2, v_2)?

Assume (x, v) hides (x_1, v_1) using (x_r, v_r):

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NP-statement hider: Example

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$$v = v_1 + \rho v_r.$$

(x, v) is equally likely to hide (x_2, v_2) if there is $(x'_r, v'_r) \in \mathcal{R}_A$ such that:

$$x_2 + \rho' x'_r = x_1 + \rho x_r$$
$$v_2 + \rho' v'_r = v_1 + \rho v_r.$$

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So we must have

$$x'_r = (\rho')^{-1}(x_1 + \rho x_r - x_2)$$
$$v'_r = (\rho')^{-1}(v_1 + \rho v_r - v_2)$$

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But is this in $\mathcal{R}_A$?

NP-statement hider: Example

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$$x = x_1 + \rho x_r$$
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(x, v) is equally likely to hide (x_2, v_2) if there is $(x_r', v_r') \in \mathcal{R}_A$ such that:

$$x_2 + \rho' x_r' = x_1 + \rho x_r$$
$$v_2 + \rho' v_r' = v_1 + \rho v_r.$$

So we must have

$$x_r' = (\rho')^{-1}(x_1 + \rho x_r - x_2)$$
$$v_r' = (\rho')^{-1}(v_1 + \rho v_r - v_2)$$

But is this in $\mathcal{R}_A$?

$$\begin{aligned} Av_r' &= A(\rho')^{-1}(v_1 + \rho v_r - v_2) \\ &= (\rho')^{-1}(Av_1 + \rho Av_r - Av_2) \\ &= (\rho')^{-1}(x_1 \rho x_r - x_2) \\ &= x_r' \end{aligned}$$

NP-statement hider: Example

Example is secure

Can Adv distinguish if (x, v) hides (x_1, v_1) or (x_2, v_2)?

Assume (x, v) hides (x_1, v_1) using (x_r, v_r):

$$x = x_1 + \rho x_r$$
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Theorem

There is a privacy preserving folding scheme for $\mathcal{L}_A = \mathrm{Im}(A)$.

(x, v) is equally likely to hide (x_2, v_2) if there is $(x'_r, v'_r) \in \mathcal{R}_A$ such that:

$$x_2 + \rho' x'_r = x_1 + \rho x_r$$
$$v_2 + \rho' v'_r = v_1 + \rho v_r.$$

So we must have

$$x'_r = (\rho')^{-1}(x_1 + \rho x_r - x_2)$$
$$v'_r = (\rho')^{-1}(v_1 + \rho v_r - v_2)$$

But is this in $\mathcal{R}_A$?

$$\begin{aligned} A v'_r &= A(\rho')^{-1}(v_1 + \rho v_r - v_2) \\ &= (\rho')^{-1}(A v_1 + \rho A v_r - A v_2) \\ &= (\rho')^{-1}(x_1 \rho x_r - x_2) \\ &= x'_r \end{aligned}$$

In general...

NP-statement hider

If there is a folding scheme for $\mathcal{L}$, $\mathcal{R}$ supports efficient random sampling, and for any three instances $(x_1, v_1), (x_2, v_2), (x, v) \in \mathcal{R}$ there are equally many ways to fold (x_1, v_1) into (x, v) as there is to fold (x_2, v_2) into (x, v), then there is an NP-statement hider for $\mathcal{L}$.

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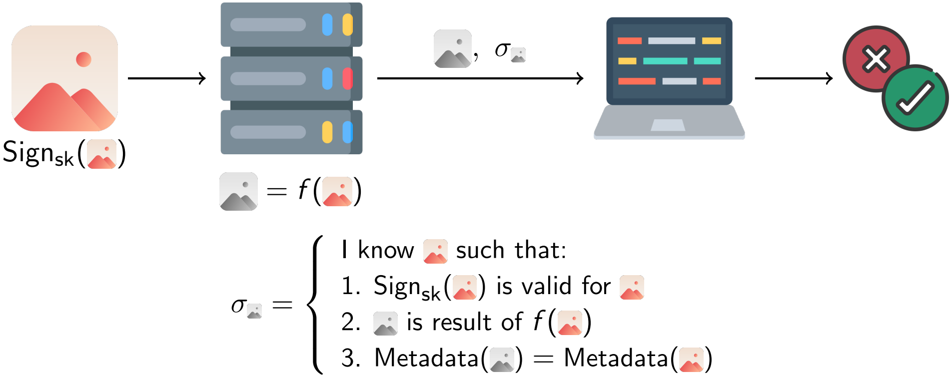
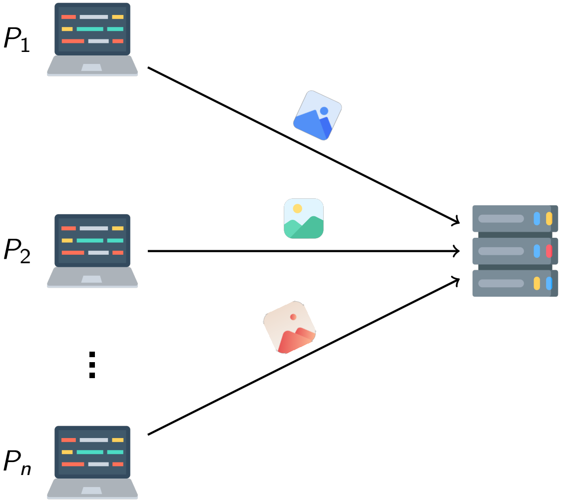
Privacy Preserving Folding Scheme

As above: There is a Privacy Preserving Folding Scheme for $\mathcal{L}$.

Returning to Images



Returning to Images



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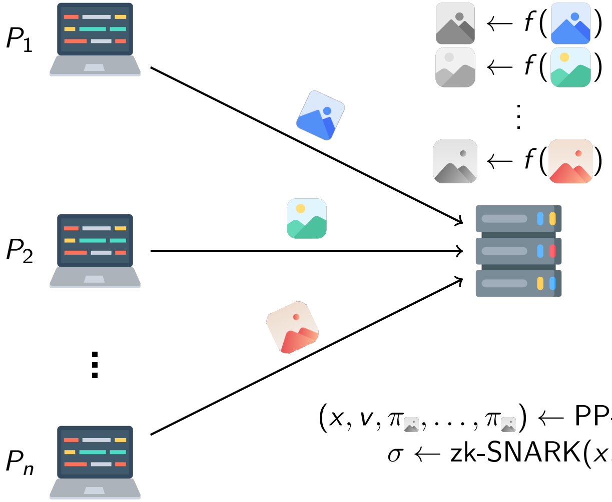
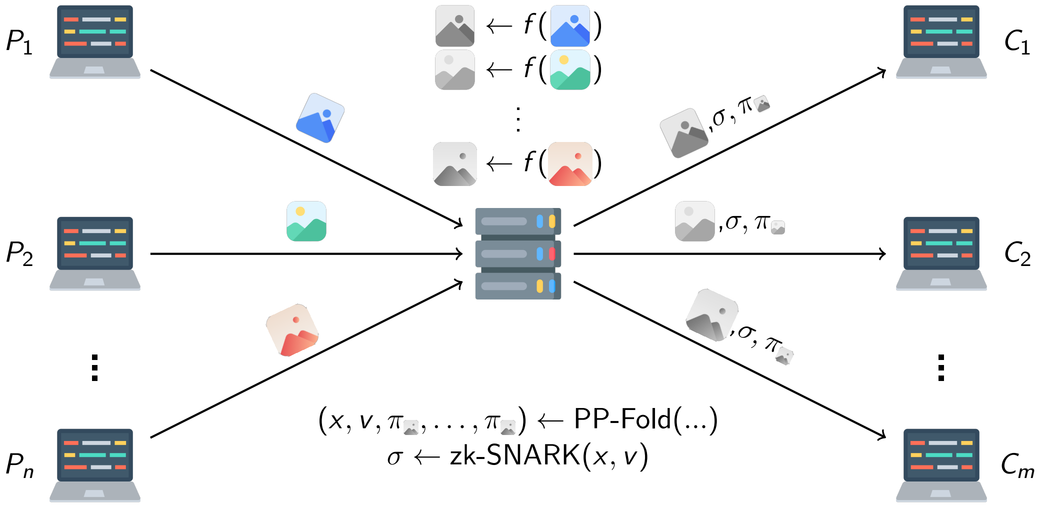


$$(x, v, \pi, \ldots, \pi) \leftarrow \text{PP-Fold}(\ldots)$$
$$\sigma \leftarrow \text{zk-SNARK}(x, v)$$

Returning to Images



Thank you for listening.

Questions?

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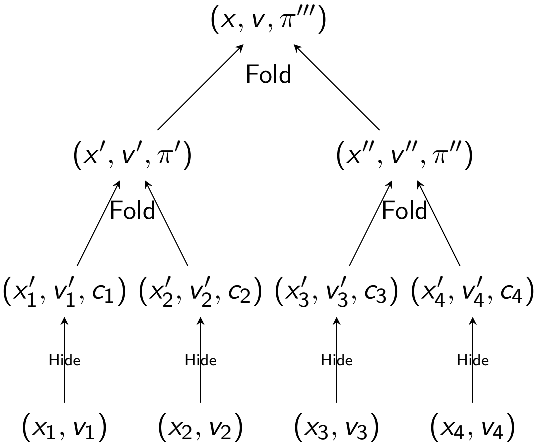
Privacy Preserving Folding Scheme [B**E**24]

$$
\begin{tikzcd}
 & & & (x, v, \pi''') & & & \\
 & (x', v', \pi') \arrow[urr, "\mathrm{Fold}"] & & & & (x'', v'', \pi'') \arrow[ull] & \\
(x_1', v_1', c_1) \arrow[ur, "\mathrm{Fold}"] & & (x_2', v_2', c_2) \arrow[ul] & & (x_3', v_3', c_3) \arrow[ur, "\mathrm{Fold}"] & & (x_4', v_4', c_4) \arrow[ul] \\
(x_1, v_1) \arrow[u, "\mathrm{Hide}"] & & (x_2, v_2) \arrow[u, "\mathrm{Hide}"] & & (x_3, v_3) \arrow[u, "\mathrm{Hide}"] & & (x_4, v_4) \arrow[u, "\mathrm{Hide}"]
\end{tikzcd}
$$

Privacy Preserving Folding Scheme [BE24]



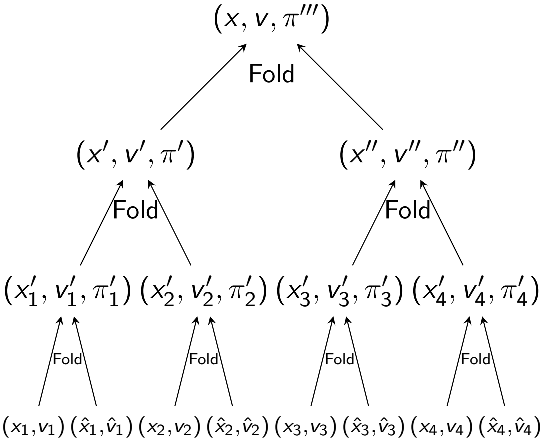


Image Signature Scheme Computation

	Computation Time	**Signature Size**

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Key generation	Same as $\mathsf{KeyGen}^{\mathsf{DS}}$	—

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	Computation Time	**Signature Size**
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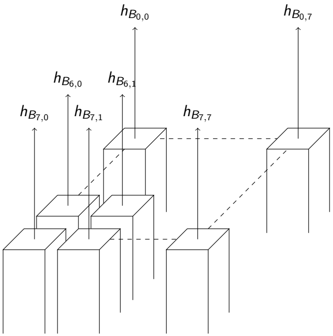
Signing	1025 hashes and time of Sign^{DS}	$\lvert S \rvert$
Compression	1025 hashes	$128\lvert H \rvert + \lvert S \rvert$
Verification	1025 hashes and time of $\text{Verify}^{\text{DS}}$	—

Image Signature Scheme Final Construction Step



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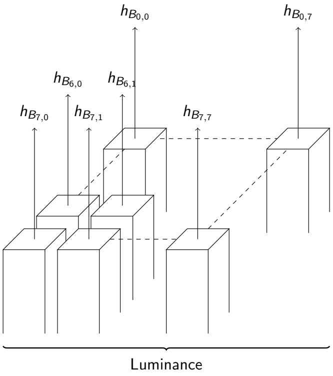
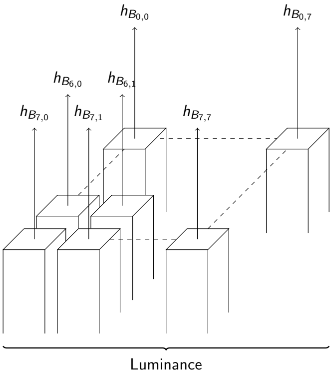
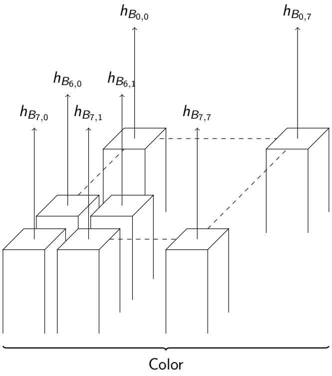


Image Signature Scheme Final Construction Step





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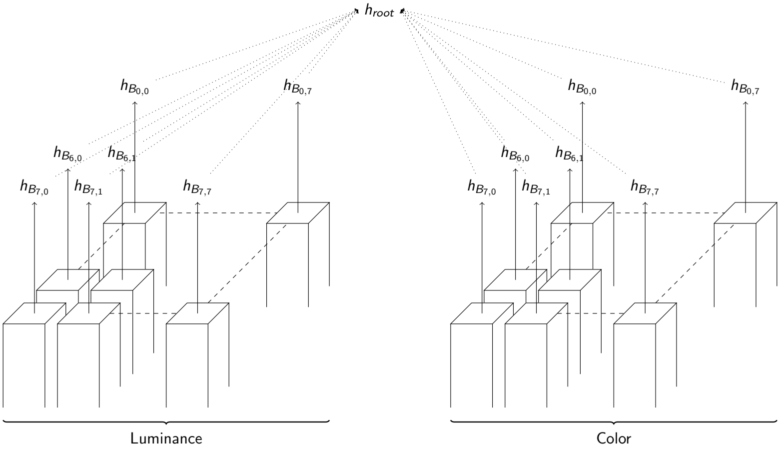

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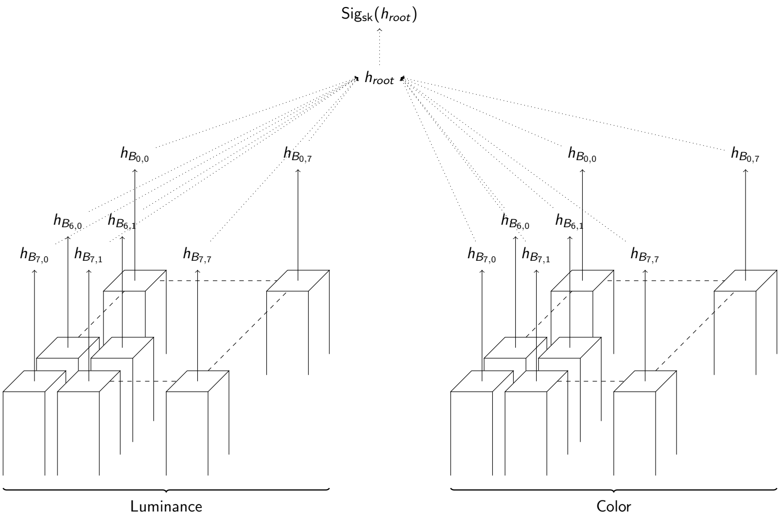

Proof of Security

Proof of Security

If $\forall k:$ i^*'s h_{root} is different from i_k's h_{root}: Forgery against DS.

Proof of Security

Else let k be such that i^* and i_k have the same h_{root}.

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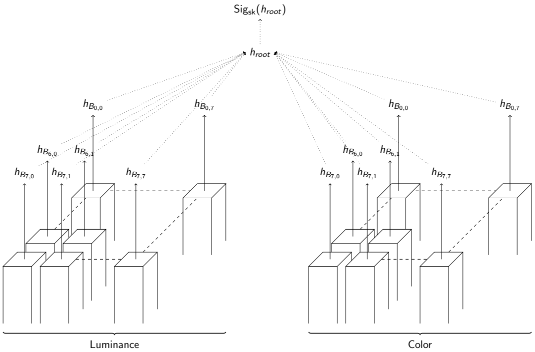

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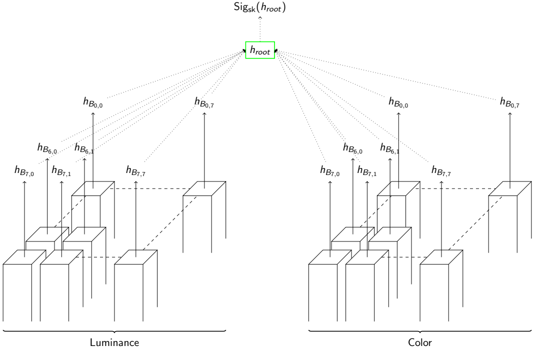

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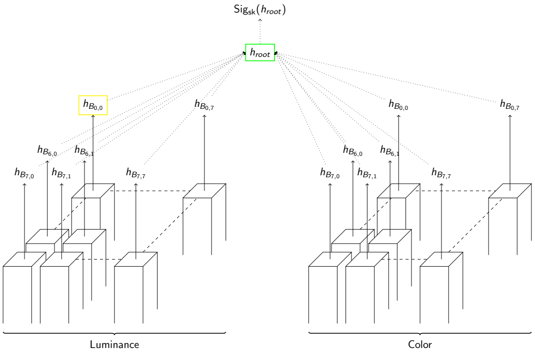

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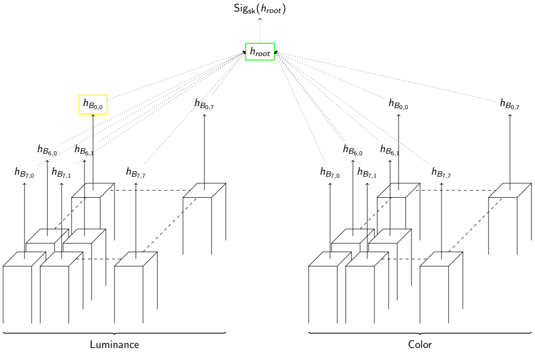

If $h_{B_{0,0}}$ is different for i^* and i^k: Collision for H:

$$H(\cdots, h^*_{B_{0,0}}, \cdots) = h_{root} = H(\cdots, h^k_{B_{0,0}}, \cdots).$$

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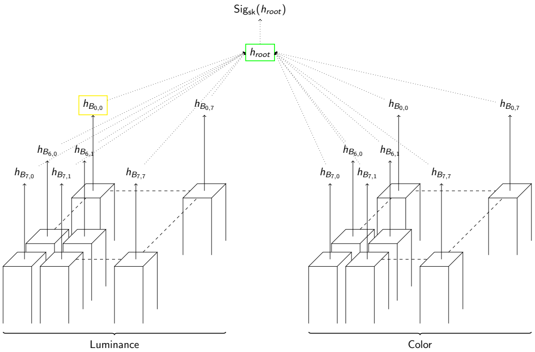

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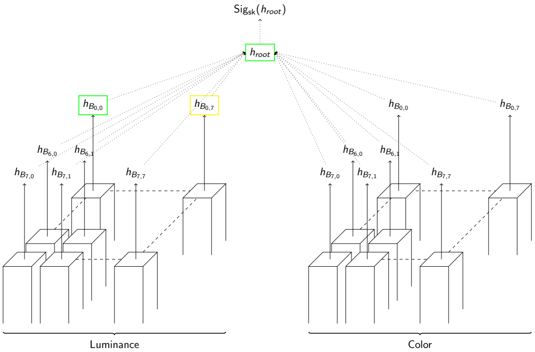

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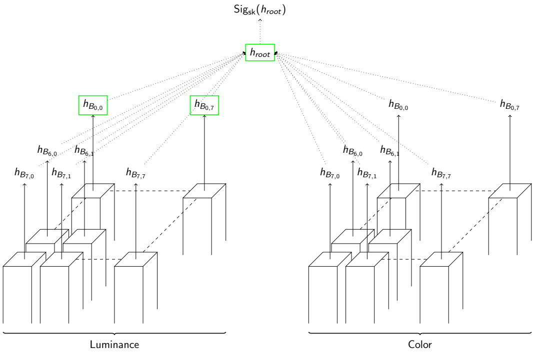

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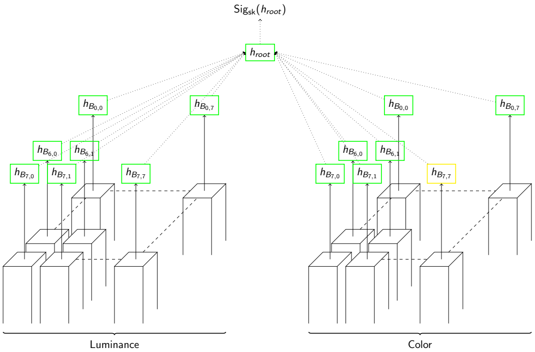

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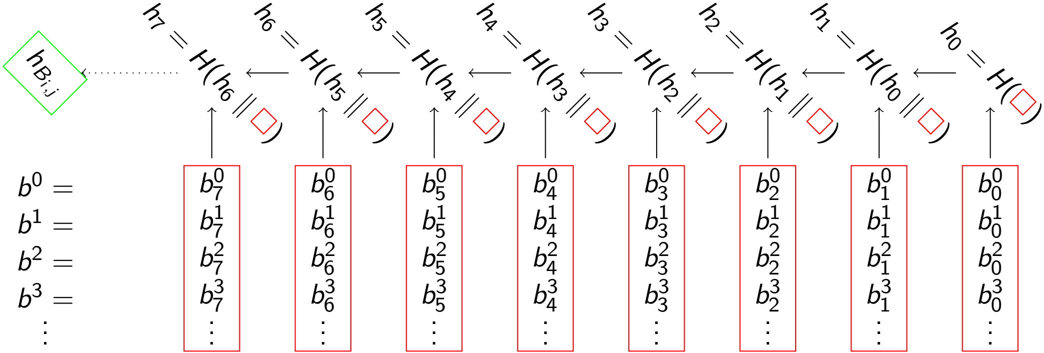
Either we have found $h^*_{B_{i,j}} \neq h^k_{B_{i,j}}$ and a collision to H.

Proof of Security

Or we go one level deeper for each $h_{B_{i,j}}$.



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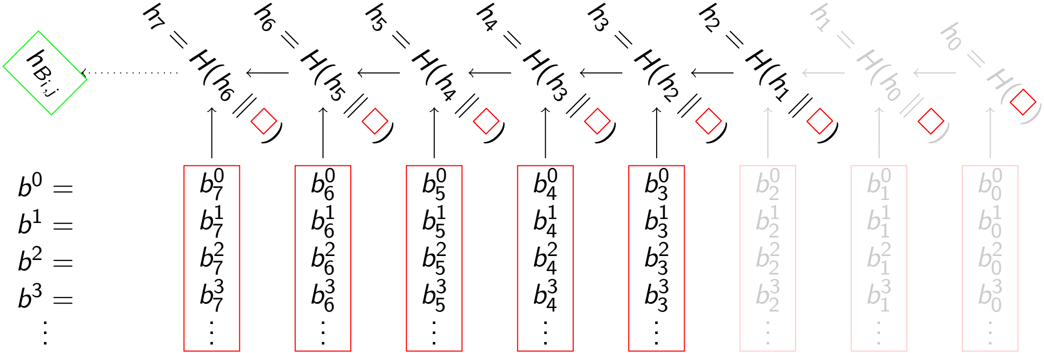

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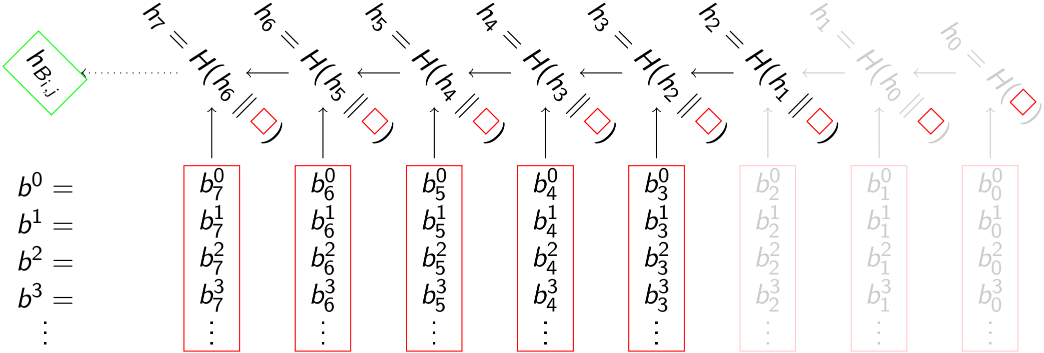

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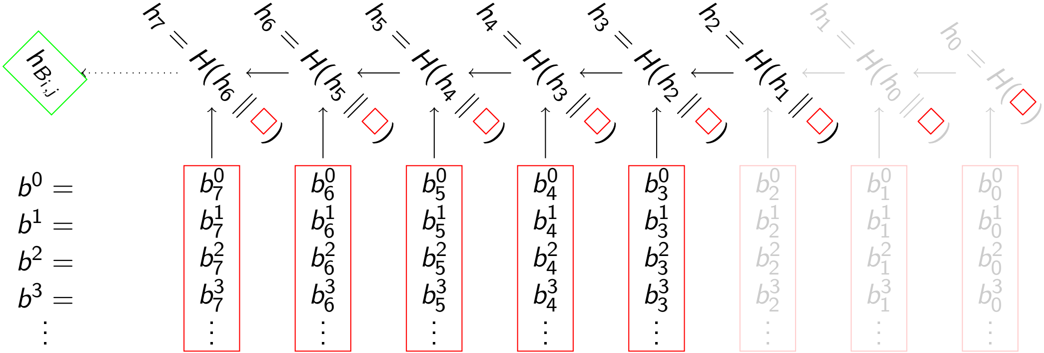

Since $i^* \notin \text{CSpan}(i_k, s_k)$ we are guaranteed tho eventually find a difference, and hence a collision for H. $\square$